Fiscal Capacity, Foreign Reserves and Lender of Last Resort

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Abstract

I develop a theory of public liquidity, fiscal capacity —the ability to collect taxes effectively— and why governments accumulate foreign reserves for liquidity purposes. I construct a liquidity framework of an economy that borrows from international markets and features a government with heterogeneous levels of fiscal capacity. Since liquidity crises arise from binding financial frictions, fiscal capacity determines the effectiveness of ex post public policies. When fiscal capacity is high, the government eliminates liquidity crises by overcoming financial frictions. When fiscal capacity is low, it cannot, forcing it to rely on second-best policies such as foreign reserves accumulation. A key mechanism behind these results is a crowding-out effect: when fiscal capacity is underdeveloped, public liquidity provision displaces private liquidity rather than expanding aggregate liquidity. In equilibrium, governments with low fiscal capacity may accumulate reserves depending on whether the expected cost of a crisis outweighs the increasing cost of reserves accumulation. I empirically test four theoretical implications using data from 44 economies between 1991 and 2019.

Keywords: Public liquidity provision, liquidity crisis, foreign reserves
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1 Introduction

There is a longstanding belief that governments are an economy's ultimate last resort during a financial crisis. A prime example of this belief is President George W. Bush's statement, "If money isn't loosened up, this sucker could go down!" to his staff while Congress struggled to agree on a \$700 billion rescue package during the Global Financial Crisis.

If one defines liquidity as a store of value or real claims,[1] then a liquidity crisis is a scenario in which the private economy cannot produce stores of value that are attractive enough to transfer resources from liquid to illiquid agents. In this scenario, as argued by Tirole (2011), sovereign debt is special because it is backed by the exclusive right of governments to collect taxes and impose non-pecuniary penalties. This allows a government to respond *after* the liquidity need materializes, while the private provision of liquidity must take place *before* it does. Thus, public liquidity provision is said to be more efficient because it does not require bearing the costs of potentially wasteful liquidity hoarding.

Yet, governments routinely engage in such preemptive liquidity policies through the accumulation of foreign reserves. Between 2000 and 2019, official holdings of foreign reserves almost tripled worldwide, rising from 5.6% to 13.4% of world GDP (Figure 1). Many monetary authorities justify this policy as a self-insurance mechanism against liquidity shortages caused by the volatility of capital flows. At the same time, the International Monetary Fund (IMF), in its Article IV consultations, considers the level of foreign reserves a critical element in evaluating a country's economic and global stability.

Although apparently at odds with the literature on liquidity, incurring the cost of foreign reserves accumulation has been widely regarded by the international monetary system as necessary for economies to safely participate in financial globalization (Rodrik, 2006).

Why do some governments engage in foreign reserves accumulation for liquidity purposes when, in theory, they can provide liquidity ex post? What does the practice of foreign reserves accumulation tell us about public liquidity provision in general? I address these questions in

[1] See Tirole (2011)

this paper.

My main argument is that a government's power to tax must be developed. Thus, the comparative advantage of public liquidity over private liquidity stems from a government's ability to effectively raise taxes—a feature defined by Besley and Persson (2009) as fiscal capacity—and not simply from its legal right to tax.

By accounting for the importance of fiscal capacity, I reconcile the practice of reserves accumulation with the literature on liquidity provision. When fiscal capacity is sufficiently high, governments can produce liquidity à la Tirole (2011). Conversely, when fiscal capacity is underdeveloped, they cannot. If such governments want to implement liquidity programs, they must resort to second-best policies such as foreign reserves accumulation. Thus, I interpret the hoarding of foreign reserves as evidence of a binding constraint on public liquidity provision due to low fiscal capacity.

The first contribution of this paper is to present a theory of public liquidity, fiscal capacity, and foreign reserves. To do this, I construct a liquidity framework of an economy that borrows from foreign markets to finance investment opportunities. International interest rates are driven by a global financial cycle which implies that, from the perspective of domestic agents, funding costs are a random aggregate shock. I introduce into this framework a credible government with varying levels of fiscal capacity.

The lending relationship between domestic agents and foreign investors is subject to financial frictions. Specifically, I assume that domestic agents can default on their external debt by running away. This creates a wedge between the total output of an investment project, and its pledgeable return - the amount of future output that domestic agents can credibly promise to foreign investors.

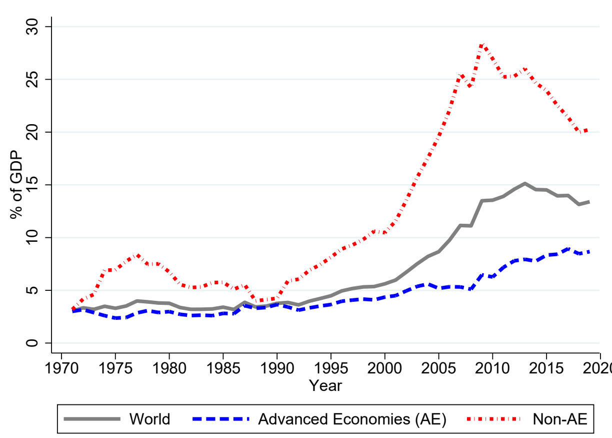
I focus on crises that would not occur in the absence of financial frictions. Thus, the gap between total and pledgeable output of investment opportunities introduces the possibility of aggregate liquidity shortages, even for states of the global financial cycle that the economy remains solvent.

When international interest rates are high, this economy's aggregate pledgeable output can become relatively unattractive to foreign investors compared to their outside options. As a result, a sudden stop of foreign lending occurs, leading to a domestic economic crisis. In the model, a credible government can potentially prevent such liquidity crisis by acting as an intermediary between foreign lenders and the domestic economy. This possible because, unlike private debt, public debt is backed up by the government's fiscal capacity.

Besley and Persson (2014) argue that greater fiscal capacity is the result of major investments in enforcement and compliance mechanisms that improve a government's ability to extract resources from its economy. In line with this view, fiscal capacity in the model determines how much tax revenue a government can collect, regardless of whether domestic agents default on foreign investors. That is, a government with high fiscal capacity can collect most of its tax revenue even in the event of default whereas a government with low fiscal capacity can only collect its tax revenue if they don't run away.

I show that fiscal capacity determines the degree to which public debt is constrained by the same financial friction as private debt. The greater the fiscal capacity, the more public debt differs from private debt, the less it is constrained by financial frictions, and the more likely the government is to prevent a liquidity crisis.

Figure 1: Official Holdings of Foreign Reserves 1970 - 2019



Source: World Development Indicators, Own Calculations

I use this framework to present four theoretical results that support the main argument. I also test empirically these theoretical implications using a sample of 44 advanced and non-advanced economies between 1991 and 2019. I summarize my theoretical and empirical findings below.

First, I show that governments with sufficiently developed fiscal capacity prevent liquidity crises without resorting to the accumulation of foreign reserves. As fiscal capacity increases, public debt becomes increasingly backed by total output rather than just aggregate pledgeable output. At some point, financial frictions stop binding sovereign debt, allowing the country to offer attractive stores of value in international markets. In contrast, public debt issued by governments with insufficient fiscal capacity remains constrained by financial frictions. These governments can offset their lack of fiscal capacity and still effectively address liquidity shortages by adopting a second-best policy such as foreign reserves accumulation. Consistent with this result, I find a negative correlation between fiscal capacity and an economy's stock of foreign reserves.

Second, I show that the costs of reserves accumulation, which rise as fiscal capacity is lower, outweigh the expected benefits for some governments with underdeveloped fiscal capacity. Thus, in equilibrium, it might be optimal to remain vulnerable against the volatility of capital flows by not accumulating reserves. Empirically, I find that the negative correlation between fiscal capacity and reserves gets stronger, in absolute value, as countries with low levels of fiscal capacity are excluded from the analysis.

Third, akin to Farhi and Tirole (2012), my model allows for multiple equilibria: an equilibrium of foreign reserves accumulation coexists with an equilibrium where private agents are the ones hoarding liquidity. However, in my model, this is only possible when fiscal capacity is underdeveloped. The reason is that governments with mature fiscal capacity produce liquidity at will, thus neither the government nor private agents have incentives to hoard liquidity preemptively. In contrast, when fiscal capacity is sufficiently low, an economy can protect itself either by accumulating reserves or when private agents self-insure against liquidity shocks. In the empirical exercises, I find, on average, a positive correlation between foreign reserves and private liquidity hoarding. Yet, this correlation is statistically smaller in countries with lower levels of fiscal capacity. I interpret this finding as suggestive empirical evidence of multiple equilibria.

At face value, my paper is not the first to relate the effectiveness of ex-post policies to a government's fiscal capacity, and foreign reserves to the lack of fiscal capacity. Bocola and Lorenzoni (2020) develop a model of a small open economy to show that liability dollarization emerges in equilibrium when domestic savers have concerns over local financial stability. In their model, ex post government policies based on either sufficient fiscal capacity or the use of foreign reserves eliminate crises, reducing incentives to borrow in foreign currency ex ante.

Having said that, the differences with Bocola and Lorenzoni (2020) are non-trivial. These authors model fiscal capacity as setting an upper bound on government's tax revenue, which I refer to as the tax revenue channel. Low levels of fiscal capacity imply fewer resources for a government to act ex post. Reserves, in their model, are a hedging instrument that boosts government resources in bad states because they are denominated in foreign currency.

In addition to the tax revenue channel, the fourth theoretical result of my paper is that I identify a second mechanism through which the lack of fiscal capacity impairs public liquidity provision: the crowding out channel.

As discussed before, fiscal capacity determines the degree to which public liquidity provision is subject to the *same* financial constraint as private liquidity provision. This means that when a government has underdeveloped fiscal capacity, public liquidity is backed up by the same pledgeable income as private liquidity. Thus, increases in public liquidity come at the expense of private liquidity. Public liquidity crowds out private funding liquidity.

Additionally, I show that in a version of the model where taxes are constrained only by the economy's solvency (with the tax revenue channel turned off), the crowding-out channel alone can explain why economies resort to foreign reserves accumulation. In contrast, the tax revenue channel, while having the crowding out effect turned off, only impairs public liquidity in states with relatively high international interest rates. Thus, I argue that the driving force in my model behind a constrained public liquidity is the crowding out effect.

In contrast to Bocola and Lorenzoni (2020), foreign currency does not play a role in my model. Reserves compensate for the lack of fiscal capacity because these are assets that do not crowd out liquidity. More precisely, reserves in my model are *foreign* in the sense that they are backed up by another economy's pledgeable output, regardless of their currency denomination. This result aligns with the common practice by central banks to hold reserves in the form of sovereign bonds issued by foreign governments.

In support of the crowding out channel, I find two empirical results. First, the correlation between public and private foreign external liabilities is positive for countries with high fiscal capacity, and non-positive in countries with low fiscal capacity. Second, the elasticity between foreign reserves and income tax revenue depends on the type of taxpayer. Whereas the elasticity is negative for individuals, it is positive for businesses. As suggested by the crowding out channel, tax structure matters.

Overall, the crowding out channel underscores that the success of an ex post public liquidity provision program rests on the government's ability to provide stores of value backed up by *new* resources, and not simply on the amount of resources available.

Literature.- The core of the theoretical framework emanates from the work of Holmström and Tirole (1998) and Farhi and Tirole (2012) which is well-summarized in Tirole (2002) and Holmström and Tirole (2011). I contribute to this literature by highlighting fiscal capacity as a limit to the effectiveness of government ex-post interventions. Tirole (2011) focuses the limits of government public liquidity on social costs and dead weight losses due to taxation and credibility concerns. Meanwhile, additional to the work by Bocola and Lorenzoni (2020), Benigno and Robatto (2019), and Farhi and Maggiori (2018) also signal out the effects of fiscal capacity on public liquidity provision. However, these papers model limited fiscal capacity as an upper bound on tax revenue whereas I identify a second mechanism, additional to the tax revenue channel, the crowding out channel. According to this second channel, what matters is not the efficiency nor the amount of of tax collection, but who is being and can be taxed. This is, to the best of my knowledge, the formalization of Tirole (2002, p.76) who briefly discusses that public debt might fail to increase aggregate liquidity when created at the expense of illiquid agents. Likewise, Calvo (2016) talks about a *Liquidity Deflation* phenomenon: increases of public liquidity *deflate* total liquidity unless it increases pledgeable output at the same time. In my environment, a government *deflates* liquidity unless they have sufficient fiscal capacity to overcome financial frictions and provide debt backed up with new pledgeable income.

This crowding out channel is also consistent with Barro et al. (2022) who build a heterogeneous-agent model with rare disasters and risk aversion to study safe asses and calculate a crowding-out coefficient of -0.5 for private bonds with respect to public bonds; and the research on the liquidity and safety services of US government debt, and how changes in government supply affect equilibrium liquidity and safety prices (Krishnamurthy and Vissing-Jorgensen, 2012, 2013). Relative to this work, I focus on the crowding out effect of liquidity provision policies.

The fiscal capacity concept used in my paper comes from state capacity literature. Research in Besley and Persson (2009), Besley and Persson (2013), and Besley and Persson (2014) shows that fiscal capacity is the outcome of both economic growth and political institutions, and countries with greater fiscal capacity are also more developed countries. Besides relating fiscal capacity to financial and economic fragility, I also contribute to this literature empirically. I show that, at least for studying the relationship between fiscal capacity and liquidity provision, a better proxy for a government's fiscal capacity is the tax revenue levied directly from individuals net income, instead of using total income tax revenue.

This paper provides a novel rationale to why countries might choose to accumulate foreign reserves. Most of the literature that has tried to understand the strong build-up of foreign reserves stocks between 2000 and 2014 has interpret this trend as developing and/or emerging economies learning the lessons from the 1990s financial crises (Feldstein, 1999), and have explicitly excluded advanced economies from their analysis. The IMF (2011) policy report summarizes why advanced economies shouldn't accumulate reserves for precautionary reasons under two main ideas: i) not exposed to sudden stops, and ii) borrow in their own currency. Consistent with this first idea, several papers need to assume an exogenous positive probability of a sudden stop or exclusion from international markets to generate a positive demand for reserves (Aizenman and Lee, 2007; Jeanne and Rancière, 2011; Calvo et al., 2013; Céspedes and Chang, 2024). In contrast, in my paper, sudden stops are one of the possible outcome of the model and, as such, they are not assumed to be specific to developing countries. This way to understand a sudden stop is consistent with Calvo et al. (2006) who argues that domestic financial vulnerabilities determine whether an initial external negative shock turns into a sudden stop or not. In my case, sufficient fiscal capacity, or sufficient reserves, can prevent the materialization of the sudden stop.

The inability to borrow in its own currency is what Eichengreen et al. (2003) refer to as *Original Sin*. This concept is linked to financial fragility because it exposes countries balance sheet to currency mismatch (Chang and Velasco, 2001). It is said that a lender of last resort that provides liquidity in foreign currency could alleviate financial instability. But to do so, it needs to accumulate dollars ex-ante to provide dollars ex-post. However, following Fischer (1999), a lender of last resort doesn't need to accumulate reserves ex-ante as long as it can come up with those resources when needed. This is a similar idea to Feldstein (1999) when believes that reserves are as useful to provide liquidity as having the ability to borrow under adverse conditions. The contribution of my paper is to underscore that the ability to attract resources ultimately depends on a country's fiscal capacity, and that the lack of fiscal capacity creates a demand for foreign reserves even in scenarios where there is no currency mismatch. The lessons of my model can be applied to both advanced as well as non-advanced economies.

My paper is closer to recent work that merges reserves accumulation with financial frictions (Dominguez, 2009; Céspedes and Chang, 2024). My paper highlights that reserves accumulation are useful only for governments that lack the fiscal capacity to overcome financial frictions without reserves in the first place. Thus, the demand for reserves emerges only when there is fiscal underdevelopment.

Recently, some interesting work has emerged on the relationship between foreign reserves and sovereign default. In a context where a government can choose the amount of reserves, and the level of sovereign debt, Alfaro and Kanczuk (2009) show that the optimal level of reserves is zero since reserves reduces the opportunity cost of defaulting, and, in equilibrium, increase the cost of debt. In contrast, Bianchi et al. (2018) find that the optimal level of reserves is positive since reserves can be used as a hedging instrument, and, as such, provide an insurance against rollover risk. Similarly, Barbosa-Alves et al. (2024) study foreign reserves management under rollover risk while Bianchi and Sosa-Padilla (2024) show that reserves are useful for macroeconomic stabilization under fixed exchange rate regimes and sovereign default. In my model I abstract from incentives to default. However, I show that limited fiscal capacity limits the ability of a government to raise resources in international markets. Future research could show how fiscal capacity and sovereign default incentives intertwined.

Lastly, my empirical exercises also contribute to the efforts of estimating the motives behind foreign reserves accumulation (Aizenman and Lee, 2007; Obstfeld et al., 2010; Ghosh et al., 2017). I show that fiscal capacity is a quantitative and robust variable behind the demand for foreign reserves, even when controlling for other variables considered previously in the literature. These results support that countries accumulate reserves for liquidity purposes.

Outline.- Section 2 presents the theoretical framework and the equilibrium without Government policy. Section 3 presents the sets of equilibria of the model with a lender of last resort. Section 4 follows with the discussion of the mechanisms of how fiscal capacity affects a government's ability to produce liquidity. I then move to the empirical exercises in Section

5. Section 6 finishes off the paper with the main takeaways as well as avenues for future research.

2 Model

2.1 Environment

I study the role of different levels of fiscal capacity for liquidity provision in an environment similar to Farhi and Tirole (2012).

The economy Consider a three period economy ($t = 0, 1, 2$) inhabited by two types of agents: banking entrepreneurs and a lender of last resort. There is a continuum of banking entrepreneurs with population normalized to 1. Agents trade, consume and invest the only perishable final good existing in this economy. Moreover, this economy is *open* in the sense that agents have access to international capital markets where they can lend or issue claims, either at period 0 or at period 1.

Foreign lenders are risk neutral and have deep pockets. They are willing to lend resources to this economy as long as they obtain, at least, the same expected return that they would get from lending at international financial markets. I denote this marginal opportunity cost between period t and period $t+1$ with γ_t. I assume that this economy is *small* such that equilibrium returns in international capital markets are not affected by decisions made by either entrepreneurs or the lender of last resort.

Banking Entrepreneurs are risk neutral agents ($U(c) = c_0 + c_1 + c_2$) that receive an endowment A of the only good in the economy at the initial period. These agents do not receive further endowments and are protected by limited liability. Entrepreneurs can consume the initial endowment at $t = 0$, they can lend it in international capital markets at the given rate, or they can use it to invest in a project.

Project Technology. Banking entrepreneurs have access to a constant return to scale investment technology (Figure 2). When i units of the perishable good are invested in the initial period, it generates a safe cash flow of πi at $t = 1$. A reinvestment of size j is required at $t = 1$ to generate any return at $t = 2$. This reinvestment cannot be greater than the initial investment i, thus, the project's size is set at $t = 0$. If j is positive, the project produces a total return of $\rho_1 j$ at $t = 2$. Whereas, if j is zero, the project is shutdown and doesn't generate any return beyond the safe cash flow.

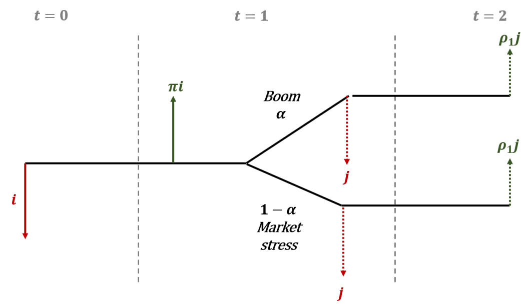


Figure 2: Project Technology - Timeline

Banking entrepreneurs cover initial investment and reinvestments either by using the liability side of their balance sheet (funding liquidity), or by using the asset side (market liquidity).[2] In this model, entrepreneurs tap on their funding liquidity by issuing short-term and long-term claims at international capital markets (private funding liquidity) or by borrowing from the lender of last resort (public liquidity). Short-term claims are backed up by projects safe cash flow while the rest of liabilities are are backed up by projects date-2 pledgeable return - More on this below. In regard of market liquidity, entrepreneurs use their initial endowment at $t = 0$, and, plausibly, any return they receive from the project or world capital markets at $t = 1$.

Moral Hazard. I introduce a friction to a project's funding liquidity by assuming that banking entrepreneurs are subject to moral hazard. At the start of $t = 2$, an entrepreneur can abscond with a fraction θ of the project's total output. If this happens, the remaining fraction $1 - \theta$ is lost. The $1 - \theta$ loss can be interpret as the cost that a banking entrepreneur needs to successfully abscond.

Aggregate Shock. As mentioned before, this economy 's funding cost is subject to the

[2]See Tirole (2011) for a further discussion on market and funding liquidity

opportunity cost in world capital markets which is random. At $t = 1$, the state of the world could either be a *boom* where γ_1 is equal to γ_1^L with probability α or it could be experiencing a stress event with γ_1 equal to γ_1^H with probability $1 - \alpha$.

Additionally, the opportunity cost and their probabilities are exogenous to any idiosyncrasies of the domestic economy. Both are modeling choices to capture a world where financial costs are driven by a Global Financial Cycle as in Rey (2015), and where funding costs are potentially relatively expensive (stress) or relatively cheap (boom). Naturally, this interpretation is consistent with $\gamma_1^L < \gamma_0 \leq \gamma_1^H$.

Assumption 1 (Project's High Return)

- $\frac{\rho_1}{\gamma_1^H} + \pi > 1 + \alpha\gamma_1^L + (1 - \alpha)\gamma_1^H$
- $\alpha(\rho_1 - \gamma_1^L) + \pi > 1$

Assumption 1 guarantees that projects have a return attractive enough for entrepreneurs to invest all their net worth even when compared to high funding costs. This assumption is straight forward: a banking entrepreneur needs to invest one unit at $t = 0$ and a second unit additional investment at $t = 1$ with an expected net cost of $\alpha\gamma_1^L + (1 - \alpha)\gamma_1^H - \pi$ which is reflected on the right hand side of Numeral 1. The left hand side states that project's total return relatively to the high funding cost is sufficient $\frac{\rho_1}{\gamma_1^H}$ to cover for the investment cost. Numeral 2, in turn, states that the expected net return of the project if no reinvestment is done under market stress is still positive at date-0.

Lender of last Resort (LOLR) is a key agent in this small economy. Following Holmström and Tirole (1998), it is the only player in this economy that has the power to audit incomes and impose non-financial penalties to banking entrepreneurs in order to collect payments. As it will be clearer below, this unique ability provides a potentially welfare improving role for a LOLR when there are financial frictions between agents that demand liquidity (banking entrepreneurs) and those that supply liquidity (foreign lenders). Importantly, LOLR actions are limited by the sphere of the domestic economy.

Fiscal Capacity parameter $\bar{\mu}$, which can take any value between zero and 1, captures the level of development of the LOLR's fiscal capacity. I view a LOLR with greater fiscal capacity as one that has made the necessary investments in enforcement and compliance, for example, such that it can collect a greater share of $\hat{R}\tau$ directly from entrepreneurs. In other words, given $\hat{R}\tau$ owed by an entrepreneur, the LOLR can collect up to fraction $\bar{\mu}$ even if the banking entrepreneur absconds. This interpretation is consistent with Besley et al. (2013) who use the share of tax revenue that is collected through income tax as a proxy for a country's fiscal capacity.

Lending Scheme. Following Bagehot (1873)'s rule, I assume that the LOLR implements an ex-post liquidity provision program to guarantee the continuation of projects regardless of the state of the world. At $t = 1$, a banking entrepreneur can ask the LOLR for a loan, τ, to finance reinvestment. In return, the LOLR collects $\hat{R}\tau$ at $t = 2$ where $\mu\hat{R}\tau$ comes directly from entrepreneurs and $(1 - \mu)\hat{R}\tau$ comes from projects. In principle, μ is a choice variable between $\left[0, \bar{\mu}\right]$. To cover this potential demand for loans at $t = 1$, the LOLR:

- Collects F_0 resources from banking entrepreneurs at $t = 0$ and invest them in international markets. Each unit that the LOLR collects at $t = 0$ is not invested in the project, and, thus, incurs in a opportunity cost ψ.
- Transfer f_1 from its market liquidity. Hence, f_1 is less or equal to $\gamma_0 F_0$.
- Issue bonds, denoted by B_1, at international markets that need to be fully redeemed at $t = 2$.

Policy Instruments comprise set $\Gamma(\bar{\mu})^s$ and depend on the LOLR's fiscal capacity. These instruments are i) $\mu \,\epsilon \left[0, \bar{\mu}\right]$ that determines how the LOLR collects payments between entrepreneurs and projects, ii) the amount of reserves accumulated at $t = 0$ (F_0), iii) the cost of public liquidity ($\hat{R}^s$), iv) the depletion of reserves (f_1^s), and v) bond issuance (B_1^s) at $t = 1$ in every state of the world.

Policy Objective The trade-off faced by an LOLR is captured by the Policy Objective Function (1). Deviating a unit of initial endowment from projects by accumulating F_0 implies giving up a marginal net return ψ. I interpret ψ, at least, as equivalent to the difference between a project's expected return and the return from lending such unit in international markets which is equal to $\frac{\rho_1}{(1-\alpha)\gamma_1^H+\alpha\gamma_1^L} + \pi - 2$ and, by Assumption 1, is strictly positive. Additionally, ψ can also capture the positive externalities valued by a LOLR associated to the initial investment scale of projects.

$$\psi F_0 + E_s\big[L(j^s)\big] \tag{1}$$

The second term reflects the expected welfare costs of partial liquidation. Loss Function $L(j^s)$ depicts, in a reduced form, losses due to rises in unemployment or increases in financial fragility, for example. This approach follows Farhi and Tirole (2012) with the purpose to underscore that an LOLR dislikes negative spillover effects of downsizing on the economy that, individually, entrepreneurs might fail to do identify. Naturally the loss function is key in the model since it is what drives the agent with the fiscal capacity to provide liquidity ex-post.

Assumption 2 (Welfare Loss Function)
Define function $L : \big[0, i\big] \to R^+$ with the following characteristics;

1. *Continuous and convex function*
2. *Non-increasing*
3. *Bounded from below by zero when $L(i) = 0$*
4. *bounded from above by a positive constant $L(0) = K$*

I assume that $L(j)$ is bounded from below at zero when full-scale reinvestment is reached $(j = i)$. That is, there are no welfare gains for reinvestment levels beyond initial scale. Additionally, $L(j^s)$ is convex to reflect that small levels of downsizing produce lower marginal losses than larger magnitudes.

Moreover, I assume that the loss function is bounded from above by a very large positive constant. If this were not the case, a LOLR would always do whatever is necessary to prevent a complete shutdown, no matter the cost, thus eliminating interesting equilibrium results. This might be a plausible description for some economies, yet some countries find it costly to insure against all crises since opportunity costs are relatively higher. This upper bound reflects the inability to do "Whatever it Takes".

2.2 Timeline and Optimal Decision Problems

The LOLR and banking entrepreneurs are the only active decision makers in the model. Foreign lenders do not have a maximization problem, but they are willing to lend to any entrepreneur as long as the expected return is, at least, equal to opportunity cost at international markets. I describe the decision process illustrated by Figure 3.

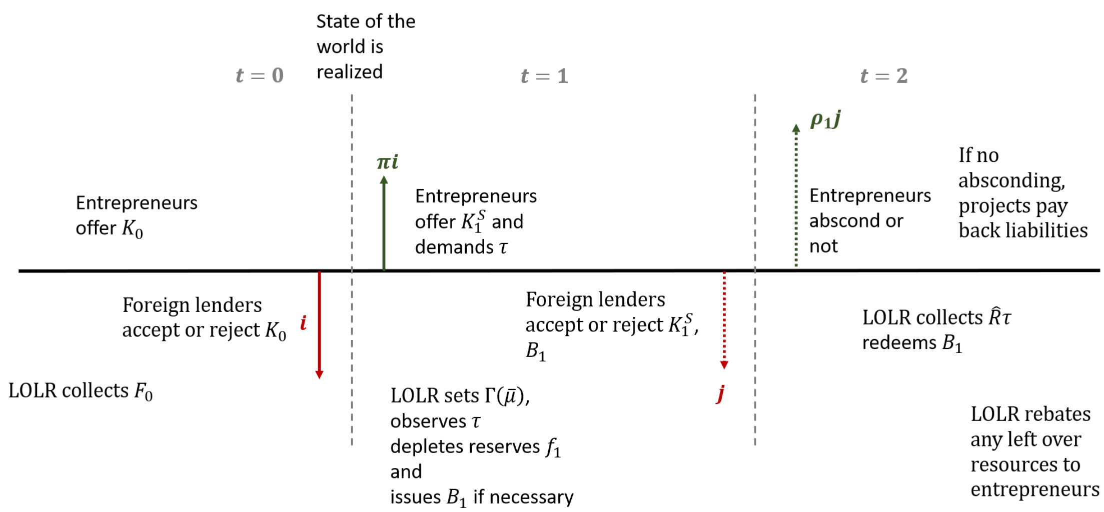


Figure 3: Model Timeline

2.2.1 Period 0 - Project's initial scale and Reserves Accumulation

At $t = 0$, the LOLR collects F_0 from the domestic economy. In turn, banking entrepreneurs offer contract K_0 to foreign investors that stipulates the initial investment scale i, the amount of entrepreneur's market liquidity to be invested M_0, and the total amount to borrow ϕ_0 from investors which is collected by issuing contingent short-term debt $(d_f^L i, d_f^H i)$ and long-term debt (l_0^L). That is, K_0 is equal to set $\{i,\ M_0,\ \phi_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}$.

A project's initial investment is covered with market liquidity and borrowing from foreign lenders. In turn, the market liquidity available for an entrepreneur at $t = 0$ is bounded by its disposable endowment $(A - F_0)$. This endowment can also be used to consume (c_0) and to lend in international markets at the initial period (x_A). The possible uses for $A - F_0$ are described by (2).

$$c_0 + M_0 + x_A = A - F_0 \tag{2}$$

Contracts K_0 must offer the same expected return than international markets to attract foreign lenders. The opportunity cost of a foreign lender at $t = 0$ is $\bar{\gamma}_0$ which is normalized to one. Thus, a project's borrowing capacity at the initial period is given by (3).

$$(i - M_0) = E_s\left[l_0^s + d_f^s i\right] \tag{3}$$

Foreign lenders expected return depends on the return offered through short-term and long-term claims. Short-term contingent claims are backed up by the safe-cash flow produced by projects at $t = 1$. Thus, an entrepreneur allocates πi in state s between foreign lenders (d_f^s) and themselves (d_e^s) as presented by (4). By offering a higher payoff d_f^s, an entrepreneur increases the amount it borrows from abroad but, by doing so, it reduces the amount of resources available to reinvest through market liquidity at $t = 1$, denoted by x_1^s, as shown by (5). As it is shown below, these resources are key to determine whether a project survives or gets shutdown during episodes of market stress.

$$d_f^s i + d_e^s i = \pi i \tag{4}$$

$$d_e^s i + x_A = x_1^s \tag{5}$$

Similar to Farhi and Tirole (2012), I focus on a contract where long-term claims are not available for stress periods. This assumption is justifiable if foreign lenders do not observe x_1^H before buying long-term claims and, thus, are reluctant to buy claims for states of the world where projects need market liquidity to achieve any continuation.[3]

Long-term claims issued in the initial period (l_0^L) are bounded by the project's long-run pledgeable output (6). Since the continuation of projects to $t = 2$ depends on the reinvestment made at $t = 1$, I assume that private and public liabilities issued in $t = 1$, denoted by l_1^L and $(1-\bar{\mu})\hat{R}\tau^L$, respectively, have seniority over l_0^L. Consequently, depending on the state of the world and decisions made at $t = 1$, there could be positive pledgeable income left to back up l_0^s.

$$l_0^L \; \epsilon \Big[0, \; \rho_0 j^L - l_1^L - (1-\bar{\mu})\hat{R}\tau^L\Big] \tag{6}$$

At $t = 0$, entrepreneurs want to maximize their expected consumption (7). To do so, it chooses non-negative set $\{c_0, x_a, K_0\}$ subject to (2), (3), (4), (5), (6) for a given F_0.

$$c_0 + E_s\Big[C_{1,2}^s - l_0^s\Big] \tag{7}$$

As said previously, the only action from the LOLR during the initial period is to collect F_0 from its domestic economy which it invests in international markets with a gross return normalized to one. Given that entrepreneurs are protected by limited liability, LOLR's minimize their policy objective (1) subject to $F_0 \leq A$ and private liquidity holdings $\{x_1^L, x_1^H\}$.

2.2.2 Period 1 - Boom or Market Stress

At the onset of $t = 1$, the aggregate shock is realized and projects produce a safe cash flow return πi which is allocated between entrepreneurs and foreign investors as determined

[3]This assumption makes the model more tractable at the cost of potentially interesting outcomes. For example, in principle, foreign lenders should be more willing to buy long-term claims for states that they anticipate that an LOLR's will provide liquidity assistance.

by K_0. The state of international markets, the amount of market liquidity in hands of entrepreneurs (x_1^s), the LOLR fiscal capacity ($\bar{\mu}$) and its savings (F_0) fully describe the state of the domestic economy at $t = 1$.

Banking entrepreneurs use x_1^s to consume immediately, to reinvest in the project M_1^s, and/or to lend at international markets to obtain x_2 at $t = 2$. Naturally, x_2 is positive only when the following inequality doesn't bind.

$$c_1^s + M_1^s \leq x_1^s \tag{8}$$

Entrepreneurs also have the option to return to world markets a second time to sell additional claims valued at $\phi_1^s j$. Importantly, I assume that, even under a stress period, the domestic economy keeps access to international markets.

Reinvestment (j^s) is financed using entrepreneur's market liquidity (M_1^s), the transfer from the LOLR (τ^s), and with foreign funds ($\phi_1^s j$). Moreover, I assume that j^s cannot be greater than the initial investment scale to capture that the scale of the model is set at $t = 0$ and it cannot be changed at $t = 1$.

$$j^s = min\{\frac{M_1^s + \tau^s}{1 - \phi_1^s}, i\} \tag{9}$$

At $t = 1$ offer foreign investors a contract $K_1^s = \{j^s,\ M_1^s,\ \phi_1^s,\ \tau^s,\ l_1^s\}$. Contract K_1^s has to be attractive enough for foreign lenders. This requires that the date-2 value of claims sold at $t = 1$, denoted by l_1^s, is at least equal to the expected return in international markets times the amount borrowed (10). Additionally, I focus on contracts that are incentive compatible such that the entrepreneur doesn't abscond. Therefore, l_1^s and τ^s are constrained by (14).

$$\gamma_1^s \phi_1^s j^s \leq l_1^s \tag{10}$$

At $t = 1$, entrepreneurs maximize their consumption at periods 1 and 2 ($C_{1,2}^s = c_1^s + c_2^s$) by choosing a non-negative set $\{c_1^s, K_1^s\}$ subject to (8), (9), (10), (14), (15), and policy set

$\Gamma(\bar{\mu})^s$.

The LOLR, simultaneously, establishes a liquidity provision program, as described above, with the objective to minimize the potential welfare losses due to partial liquidation of projects. At $t = 1$, any demand for public liquidity by entrepreneurs is covered by either issuing bonds or by depleting reserves. Naturally, f_1 is limited by the amount of reserves that were collected at $t = 0$.

$$B_1 + f_1 = \tau^s \tag{11}$$

$$f_1 \leq F_0 \tag{12}$$

2.2.3 Period 2 - Limited Pledgeability, Limited Liability, and Fiscal Capacity

At $t = 2$, entrepreneurs gross income consists of the project's net worth (n_2), any additional return from international markets (x_2), and any resources rebated back by the LOLR ($T_2 = \gamma_1^s(F_0 - f_1) + \hat{R}\tau - \gamma_1^s B_1$).

A project's net worth n_2 is equal to the difference between a its assets ($\rho_1 j$) and liabilities. At $t = 2$, liabilities consist of claims owed to foreign investors (l_f) which are equal to the sum of long term claims sold at international markets at $t = 0$ (l_0) and claims issued at $t = 1$ (l_1) and, potentially, any debt owed to the LOLR ($(1-\mu)\hat{R}\tau$). Notice that a project's assets depend exclusively on the level of reinvestment made in the previous period (j).

$$n_2 = \rho_1 j - l_f - (1-\mu)\hat{R}\tau$$

At the beginning of $t = 2$, banking entrepreneurs decide whether to abscond with share θ of project's total return or not. I focus on equilibria where they choose to not abscond.

The credibility behind the promise to abide rests on contracts K_0 and K_1 satisfying an incentive compatibility constraint. When (13) holds, project's net worth (n_2) is sufficiently high such that it is in the benefit of entrepreneurs to follow through with claims and not abscond.

$$n_2 \geq \theta\rho_1 j \tag{13}$$

Given projects balance sheet, satisfying (13) implies that projects pledgeable income at $t = 2$ is equal to $\rho_0 j = \rho_1(1-\theta)j$, and, more importantly, that it bounds both the value of long term claims sold in international markets and the payment to the LOLR that is collected directly from the project (14).

$$\rho_0 j \geq l_f + (1-\mu)\hat{R}\tau \tag{14}$$

Assumption 3 establishes that projects are liquidity constrained in some states of the world.[4] Numeral 1 establishes that a project's marginal pledgeable income when reinvestment only happens in boom states is not sufficient to cover the opportunity cost of foreign lenders at the initial period ($\gamma_0 = 1$). This is a necessary assumption for the initial investment scale is determined, otherwise, entrepreneurs con borrow infinite amounts of resources.

Additionally, Numeral 1 also implies that safe cash flow is not enough to finance solely full-scale reinvestment at $t = 1$ ($1 > \pi$). However, it is enough when paired with pledgeable income (Numeral 2). Thus, full-scale reinvestment is feasible during stress periods through a combination of market and funding liquidity.

Assumption 3 (Liquidity Constrained Projects)

1. $1 > \pi + \alpha(\rho_0 - \gamma_1^L)$

2. $\frac{\rho_0}{1-\pi} \geq \gamma_1^H$

3. $\gamma_1^H \geq 1 > \rho_0$

4. $min\{\pi, \rho_0\} \geq \gamma_1^L$

[4] If this were not the case, there is no need for liquidity management by entrepreneurs since it could always *finance-as-you-go* any reinvestment - See Tirole (2011)

Assumption 1 paired with numeral 1 of Assumption 3 implies that projects are socially valuable even in a stress episode ($\frac{\rho_1}{\gamma_1^H} > 1$). Thus, their continuation is warranted at full scale. However, I assume that projects are liquidity constrained at the initial period and during market stress events (Numeral 3).

In contrast, I assume that the funding cost during a boom state is relatively small ($min\{\pi, \rho_0\} \geq \gamma_1^L$). In fact, so small that, at in this state of the world, projects can self-finance any reinvestment ($\rho_0 > \gamma_1^L$).

Due to financial frictions, even when projects are capable of generating sufficient return to cover financing costs (Assumption 1), they can potentially shutdown because they don't produce enough *pledgeable* liquidity. In this environment, limited pledgeability is the symptom but moral hazard is the culprit for a project's inability to be liquid in all states of the world.

Entrepreneurs consumption at $t = 2$ is equal to the sum of projects net worth, any additional income derived from lending at world markets ($x_2^s = \gamma_1^s(x_1^s - M_1^s - c_1^s)$), any transfers from LOLR (T_2) net of what the LOLR collects directly from entrepreneurs.

$$c_2^s = n_2^s + x_2 + T_2 - \mu\hat{R}^s\tau^s \tag{15}$$

Since c_2^s cannot be negative, limited liability sets an additional upper bound on the total amount of resources that an LOLR can extract form its economy in the last period.The total return of a project plus total aggregate savings have to be, at least, enough to redeem long term claims sold to foreign lenders and to pay back fully the LOLR as well - (16). Unlike (14), this result is independent of an LOLR's fiscal capacity (μ).

$$x_2 + T_2 + \rho_1 j \geq l_f + \hat{R}\tau \tag{16}$$

Fiscal Capacity is a key feature for whether public liquidity provision alleviates moral hazard or not. The key assumption is that the share of what an LOLR charges directly to a banking entrepreneur is collected even if it decides to abscond. As a result, limited pledgeability sets a limit on what an LOLR collects from projects, $(1-\mu)\hat{R}\tau$, but not on what it can collect directly from entrepreneurs ($\mu\hat{R}\tau$). Clearly, if LOLR can choose μ, then it is weakly optimal to set μ equal to $\bar{\mu}$ since it maximizes the share of the revenue that is not limited by pledgeable income. I assume that this holds hereafter.

A better way to see the importance of $\bar{\mu}$ is by comparing extreme values: for a LOLR with fully developed fiscal capacity ($\bar{\mu}=1$), $\hat{R}\tau$ is bounded by Equation 16 while, for a LOLR with $\bar{\mu}$ equal to zero, $\hat{R}\tau$ is bounded by Equation 14, which, since $\rho_1 > \rho_0$, is strictly lower.

Upper bounds on $\hat{R}\tau$ matter because these resources are collected to redeem bonds issued at $t=1$, and, as a result, set a limit on the amount that can be issued. However, recall that the LOLR also collects F_0 at $t=0$ which can be used to cover some share of τ reducing the amount of government bonds that need to be issued in the first place. This idea is depicted in the model through foreign lenders participation constraint (17).

$$\gamma_1^s\tau - \gamma_1^s F_0 \leq \hat{R}\tau \tag{17}$$

Foreign lenders buy bonds from LOLR as long as the share of liquidity demand not covered with reserves valued at $t=2$ ($\gamma_1^s\tau - \gamma_1^s F_0$) is less or equal to $\hat{R}\tau$. It is worth mentioning that to derive this condition it is not necessary to assume that reserves are used as collateral to these bonds. In fact, it is sufficient to assume that reserves can be used to cover a share of τ and that an LOLR can only use up to F_0 to do so. This result is relevant because it implies even if I assume that a central bank controls reserves and that bonds are issued by a central government, fiscal capacity affects the decision to accumulate reserves as long as the central bank also dislikes the liquidation of projects.

$$\bar{R}(\tau, F_0) \geq \begin{cases} \gamma_1^s & \text{if } \tau = 0 \\ max\{0, \gamma_1^s[1 - \frac{F_0}{\tau}]\} & \text{if } \tau > 0 \end{cases} \tag{18}$$

I define function $\bar{R}(\tau, F_0)$ as the minimum cost of public liquidity for bonds to be redeemable. As long as $\hat{R}$ is equal or greater to $\bar{R}(\tau, F_0)$, (17) is satisfied and LOLR's bonds are bought by foreign lenders.

For positive values of τ, $\bar{R}(\tau, F_0)$ is a non-increasing function with respect to F_0 with an upper bound equal to the return observed in international markets when F_0 are zero. Accumulating reserves, then, allows an LOLR to offer its domestic economy a financing source that is less expensive than foreign lenders opportunity cost.

2.3 Brief discussion of modeling choices

The basic setup of this model has a similar flavor to Farhi and Tirole (2012). I discuss in this section the main differences with respect to my model, and I finish off with some comments about foreign reserves.

First, I introduce an LOLR that can have different levels of fiscal capacity. This is intended to capture different type of governments and its effect on public liquidity provision. My modeling approach is more general to what is usually observed in the liquidity literature, and papers such as Holmström and Tirole (1998) and Farhi and Tirole (2012) are examples where governments have a fully developed fiscal capacity which is the case when $\bar{\mu}$ is equal to 1 in my model.

Second, fiscal capacity determines to what extent is the LOLR subject to the same financial frictions as private agents. Hence, in contrast to Farhi and Tirole (2012), I explicitly model a financial friction that creates a wedge between total and pledgeable income. To do so, I assume that entrepreneurs can abscond with a share of the total return of the project. Meanwhile, Holmström and Tirole (1998) model this gap as the result of entrepreneurs choosing different effort levels.[5] At the end, both wedges come from the possibility of capturing a private benefit. What is key for my model is that such private benefit exists and that the level of fiscal capacity determines how much of it can be collected by the LOLR.

Third, Farhi and Tirole (2012) mainly study a liquidity provision program involving reductions in the economy's interest rate. Instead, I assume a liquidity program that consists of transfers between a LOLR and its domestic agents at a cost. Rey (2015) put into question monetary independence for economies participating in international capital markets when financing costs are driven by a global financial cycle. Thus, a liquidity provision policy has the added advantage, given my research question, that it is an instrument available in economies with different types of LOLRs, and, thus, better suited for comparability.

Four, the liquidity shock comes from random international funding costs whereas, in Farhi and Tirole (2012), the liquidity shock is modeled as a potential need for reinvestment. Although, at first, these might seem as two different modeling choices, they are not. As discussed by Tirole (2011), what creates the demand for liquidity is the inability to *finance as you go* outlays in some states of the world. Thus, from the perspective of liquidity management, the *no crisis state* in Farhi and Tirole (2012) is equivalent to the *boom state* in my model.

I interpret the LOLR in this economy more as a crisis lender/manager which, as Fischer (1999) discusses, doesn't necessarily need to be a central bank. In the model, the LOLR is closer to a general government. This wider interpretation implies, for example, that this model's definition of reserves include external assets that are not in direct control of central banks (i.e. sovereign wealth funds).

Another difference is that the IMF's definition for reserves requires these assets to be denominated in foreign currency.[6] Reserves, as I discuss further below, are foreign in my model because they are assets backed up by pledgeable output from other economies. This is con-

[5] See Holmström and Tirole (2011) or Tirole (2011) for different ways to model an agency wedge between total and pledgeable return.

[6] See Chapter 6 of the IMF's Balance of Payments and International Investment Position Manual - Sixth Edition

sistent with the fact that central banks usually have reserves invested in foreign government bonds.

Lastly, in this environment, an LOLR's is indifferent between issuing debt or depleting its stock of reserves to cover τ. This might seem as a strong assumption. One could add a dead-weight cost to bond issuance which would push LOLR's to fully deplete its stock of reserves before considering issuing any new debt. However, some countries have been reluctant to use their reserves as the primary tool to provide liquidity, even during severe crisis.[7] Basu et al. (2018) argues that this reluctance can be explained because reserves are an instrument with a zero lower bound.[8] In turn, Chamon et al. (2019) suggest that most of the benefit of reserves comes from their role *off equilibrium*, and, as a result, they are almost never used. I abstract from the analysis of reserves management and fiscal capacity and leave this question open for future work.

2.4 Laissez Faire Equilibrium

Throughout this paper, I focus on finding Subgame Perfect Nash Equilibria where entrepreneurs don't abscond and the LOLR issues safe bonds to provide liquidity ex-post successfully. This type of equilibrium has the advantage that agents strategies are *time-consistent*. I start with the equilibrium where there is no LOLR liquidity provision policy.

Definition 1 (Laissez Faire Equilibrium (LFE))
A Laissez Faire Subgame Perfect Nash Equilibrium where banking entrepreneurs' don't abscond is characterized by the following strategy profile

- *Date-2: Entrepreneurs' don't abscond*
- *Date-1:* $\{c_1^s, K_1^s\}_{L,H}$ *solve entrepreneurs date-1 problem*
- *Date-0:* $\{c_0, x_A, K_0\}$ *solve entrepreneurs problem at the initial period*

I present the full derivation of entrepreneurs optimal behavior in a laissez faire environment in the Appendix (B.1). However, at this point, it is worth highlighting that the

[7] See IMF (2011)

[8] A country could run out of reserves

driving force behind credit rationed agents is the trade-off between initial investment scale and insurance. To see this, note that optimal investment is given by

$$i = A\kappa(\bar{x}_1^L, \bar{x}_1^H)$$

where $\kappa(\bar{x}_1^L, \bar{x}_1^H)$ is project's equity multiplier as a function of how much entrepreneurs choose to hold liquidity for each state.[9] In turn, the equity multiplier is equal to

$$\kappa(\bar{x}_1^L, \bar{x}_1^H) = \frac{1}{1 - \pi - \alpha(\rho_1 - \gamma_1^L) + \alpha\bar{x}_1^L + (1-\alpha)\bar{x}_1^H}$$

which is a decreasing function with respect to $\{x_1^s\}_{L,H}$, always positive due to Assumption 3, and greater than 1.

The equity multiplier reflects directly the trade-off between insurance and investment scale. By hoarding greater levels of x_1^S, an entrepreneur increases the continuation level of projects at $t = 1$ but, in turn, it sacrifices initial investment scale.

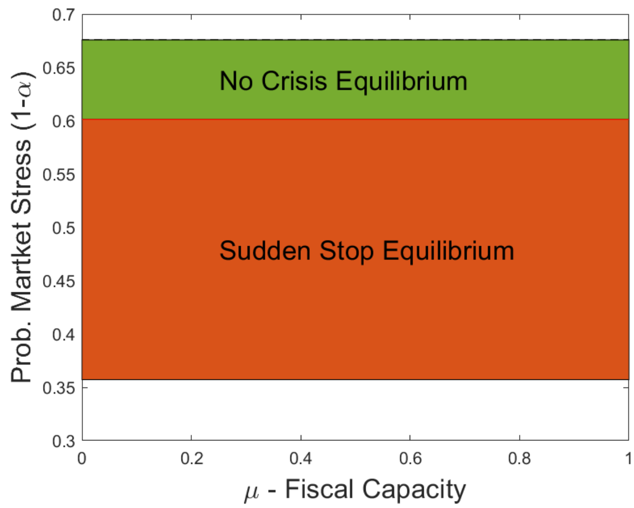
As a result of this trade-off, there are two types of Laissez Faire Equilibria (LFE) in this model depending on parameter values. The full description and proof of the LFE can be found in the technical appendix.

Figure 4 shows the two equilibria for feasible values of pair $\{(1-\alpha), \bar{\mu}\}$ for a given numerical parametrization of the model.[10] In the *No Crisis Equilibrium*, entrepreneurs optimally hoard liquidity to guarantee the continuation of projects during every state of the world. More specifically, entrepreneurs choose x_1^H to be $i\big[1 - \frac{\rho_0}{\gamma_1^H}\big]$ which is the minimum amount of market liquidity to complement with the maximum possible funding liquidity $(i\frac{\rho_0}{\gamma_1^H})$ to reach full-scale reinvestment.

[9] As is discussed in Appendix B.1, $\bar{x}_1^S$ denotes the amount of liquidity holdings per unit of investment (x_1^s/i)

[10] This numerical example is used solely for illustration purposes of the workings of the model. Parameter values are set as follows $\{A = 1, \rho_1 = 2, \theta = 0.6, \gamma_1^H = 1.4, \gamma_1^L = 0.1, \pi = 0.55, L(j) = (i-j), \psi = \frac{\rho_1}{(1-\alpha)\gamma_1^H + \alpha\gamma_1^L} + \pi - 2\}$

Figure 4: Laissez Faire Equilibrium



Proposition 1 (No Crisis - LFE)
In a No Crisis Equilibrium, an entrepreneur loads up in liquidity up to $i\left[1-\frac{\rho_0}{\gamma_1^H}\right]$ *which allows it to complement with funding liquidity and continue at full-scale when a stress episode is realized*

In contrast, in the *Sudden Stop Equilibrium*, entrepreneurs don't hoard liquidity at all ($x_1^H = 0$) which means that, if a stress episode happens, projects are forced to shutdown. I characterize this last equilibrium as *Sudden Stop* because the domestic economy is unable to attract foreign lending when funding costs are high. However, this is not because international markets are unwilling to lend to entrepreneurs but, instead, it is due to its inability to provide sufficient pledgeable income to attract expensive resources from abroad.

Proposition 2 (Subject to Sudden Stops - LFE)
Define ω *as follows*

$$\omega = \frac{\pi + \rho_0 \frac{\gamma_1^H - 1}{\gamma_1^H} - \gamma_1^L}{1 + \rho_0 \frac{\gamma_1^H - 1}{\gamma_1^H} - \gamma_1^L}$$

In a Laissez Faire environment, a market stress event turns into a Sudden Stop if $(1-\alpha) \leq \omega$

Proof In the Appendix ∎

Proposition 2 states that the existence of one equilibrium or the other depends on whether the probability of the stress period is higher or lower than a threshold. Entrepreneurs', then, hoard liquidity when the stress event is not *rare*. This threshold corresponds to red line in Figure 4. Once again, this result highlights that partial insurance is optimal in these type of models as argued by Holmström and Tirole (1998).

Since the decision to hoard depends on the trade-off between investment scale and continuation, then, not surprisingly, banking entrepreneurs don't hoard any liquidity for booms in both type of equilibria. This decision follows from modeling booms as states where projects can finance as they go their reinvestments ($\rho_0 > \gamma_1^L$). As such, there is no point in incurring costly liquidity hoarding.

Results of the LFE suggest that an LOLR's welfare improving role is warranted for *rare* stress periods. I turn now to equilibria with the intervention of an LOLR.

3 Equilibria with a Lender of Last Resort

Naturally, the results of the model when an LOLR is present hinges on how banking entrepreneurs respond. In Appendix B.6 I derive the optimal behavior of banking entrepreneurs, let me briefly present here the intuition behind this behavior.

Replacing (15) and (10) with equality, an entrepreneur's objective function at $t = 1$ can be written as follows

$$\left[1 - \gamma_1^s\right]c_1^s + \left[\rho_1 - \gamma_1^s\right]j^s + \gamma_1^s x_1^s + \left[\gamma_1^s - \hat{R}\right]\tau$$

Hence, for all feasible $\hat{R}$ considered, demanding a loan directly from the LOLR has a direct and an indirect benefit. If $\hat{R} < \gamma_1^H$, τ is marginally less expensive than ϕ^1 and M_1 which increases entrepreneurs marginal payoff (*price benefit*). This effect is reflected directly in the payoff function. Moreover, even when $\hat{R} = \gamma_1^H$, a unit of τ loosens more (14) relative to a unit of ϕ_1 as long as $\bar{\mu}$ is not zero. This allows projects to attract more liquidity through their liabilities *(indirect effect)*.

However, this indirect effect is only valuable in states of the world when γ_1^s is greater than

ρ_0 since a binding (14) prevents greater reinvestment. If not, public liquidity looses a comparative advantage. Consequently, in case of a boom, banking entrepreneurs are indifferent between private or public funding when $\hat{R}$ is equal to γ_L. This holds because projects reach full-scale reinvestment borrowing from foreign lenders. Thus, without loss of generality, I focus in an equilibrium where τ^L is equal to zero.

This is not true during a market stress. To illustrate the role that fiscal capacity has through a LOLR's *indirect effect* in providing liquidity, I abstract from reserves accumulation for the moment (I set $F_0 = 0$). Thus, for now, $\hat{R}$ is equal to γ_1^H.

The indirect effect of LOLR liquidity provision is positive for $\bar{\mu}$ such that

$$(1 - \bar{\mu})\gamma_1^H \leq \rho_0$$

That is, fiscal capacity is sufficiently developed to overcome moral hazard to some degree by having access to some share of entrepreneurs' private benefit.

$$(1 - \mu_A)\gamma_1^H = \rho_0 \tag{19}$$

The threshold at which fiscal capacity is *sufficient*, denoted as μ_A, is given by (19). This happens when the marginal cost of public liquidity on total pledgeable income is equal to pledgeable income per unit of reinvestment. Thus, one unit of τ increases, at least, as much total pledgeable income as it increases its cost.

I denote the group of LOLR that belong to interval $\left[\mu_A, 1\right]$ as Mature. Interestingly, there is no need to have a fully developed fiscal capacity to be mature in this environment. Instead, it suffice to have just enough to compensate for the wedge between the demand for liquidity and pledgeable income. In fact, note that μ_A is equal to the amount entrepreneurs hoard per unit of investment in a NO Crisis LFE.

For a LOLR that is mature, I find the Mature Fiscal Capacity Equilibrium. Proposition 3 summarizes the main characteristics of this equilibrium[11] while Figure 5 illustrates equilibria when $\hat{R} = \gamma_1^H$ for different values of $\{(1-\alpha), \bar{\mu}\}$. Similar to the No Crisis LFE, the small open economy achieves full-scale reinvestment and eliminates the possibility of a Sudden Stop in a ME. However, in contrast to the No Crisis LFE, there is no holding of private liquidity. Therefore, in an episode of market stress, any borrowing by this economy is done through the intermediation of the LOLR.

Proposition 3 (Mature Fiscal Capacity Equilibrium - ME)
Whenever $\bar{\mu} \in \left[\mu_A, 1\right]$, and Assumptions 1 and 3 hold, the following characterizes the Mature Fiscal Capacity Equilibrium (ME)

- *Entrepreneurs expect $\hat{R}$ equal to γ_1^S in each state and don't hoard liquidity for neither $\{x_1^s = 0\}_{L,H}$*
- *Initial investment i is equal to $\frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)}$*
- *The LOLR doesn't accumulate reserves ($F_0 = 0$) at $t = 0$*
- *If a boom materializes, entrepreneurs reinvest at full-scale using funding liquidity*
- *If a stress event materializes, banking entrepreneurs demand i to the LOLR who issues i abroad, entrepreneurs reinvest at full-scale*
- *At $t = 2$, after a market stress, LOLR collects $\gamma_1^H i$ to redeem bonds while entrepreneurs finally consume $(\rho_1 - \gamma_1^H)i$*

Note that the existence of the ME is independent of the probability of a crisis. That is, when a mature LOLR provides assistance, it eliminates the need to hoard liquidity even for probabilities that are relatively high or which in a LFE would be a NO Crisis Equilibrium. The reason is that a Mature LOLR intervention, in practice, *completes* markets by overriding financial frictions. There is no point to insure against a stress event at the cost of investment scale if a LOLR can provide liquidity at any time.

For LOLR that are not mature ($\bar{\mu} \leq \mu_A$), there exist, once again, two equilibria whose existence depends on the probability of a market stress. Proposition 4 summarizes the main characteristics of this case.

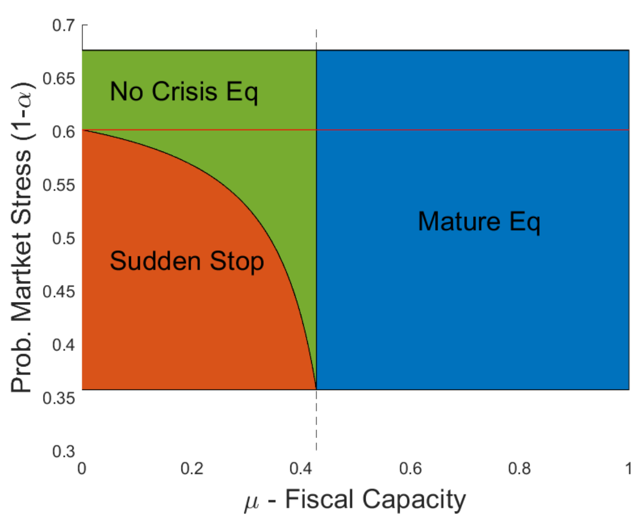
[11] I refer the reader to the appendix for the proof and complete set of strategies

Proposition 4 (No Reserves Equilibria)
In a environment with LOLR intervention but no reserves accumulation, as long as $\bar{\mu} \,\epsilon\, \big[0, \mu_A\big[$*, Assumptions 1, and 3 hold, and entrepreneurs expect* $\hat{R}$ *equal to* γ_1^H

1. *if* $(1-\alpha) \leq \omega(\bar{\mu}, \gamma_1^H)$ *there is a* ***Sudden Stop Equilibrium - No Reserves*** *where*
   - *Entrepreneurs expect* $\hat{R}$ *equal to* γ_1^S *in each state and don't hoard liquidity for neither* $\{x_1^s = 0\}_{L,H}$
   - *If a stress event materializes, banking entrepreneurs don't have enough market liquidity so the LOLR can issue bonds*
   - *The economy can't borrow and projects shutdown*
2. *if* $(1-\alpha) > \omega(\bar{\mu}, \gamma_1^H)$ *there is a* ***No Crisis - Private Hoarding Equilibrium*** *where*
   - *Entrepreneurs expect* $\hat{R}$ *equal to* γ_1^S *in each state and hold liquidity only for stress episodes equal to* $x_1^H = i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$
   - *If a stress event materializes, banking entrepreneurs borrow* $\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H} i$
   - *The economy borrows through LOLR who issues* B_1
   - *Banking Entrepreneurs reinvest at full-scale*

Consequently, unlike Mature LOLR, liquidity provision backed up by lower fiscal capacity fails to eliminate the existence of a Sudden Stop.

Figure 5: No Reserves and Mature Equilibria



Having said that, I define function $\omega(\bar{\mu}, \hat{R})$ in (20). When evaluated at γ_1^H, this function determines the threshold that determines the existence of each equilibrium presented by Proposition 4. Hence, this function establishes the probability threshold at which the economy shifts from a No Crisis to a Sudden Stop Equilibrium for policy pairs $\{\hat{R}, \bar{\mu}\}$ such that $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$.

$$\omega(\bar{\mu}, \hat{R}) = \frac{(\rho_1 - \rho_0)\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] + \left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right]\left[(\rho_0 - \gamma_1^L) - (1-\pi)\right]}{(\rho_1 - \rho_0)\left[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] + \left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right](\rho_0 - \gamma_1^L)} \tag{20}$$

Note that threshold ω from Proposition 2 is equal to $\omega(\bar{\mu}, \hat{R})$ when evaluated at $\{0, \gamma_1^H\}$. This highlights that the intermediation of a LOLR who has no fiscal capacity nor the ability to accumulate reserves cannot play a welfare improving role by intermediation between lenders and borrowers.

In contrast, the limit of $\omega(\bar{\mu}, \hat{R})$ as $\bar{\mu}$ tends to μ_A converges to

$$\frac{(\rho_0 - \gamma_1^L) - (1-\pi)}{(\rho_0 - \gamma_1^L)}$$

which is the minimum probability of a crisis that is consistent with Numeral 1 of Assumption 3. Therefore, as fiscal capacity gets closer to μ_A, the smaller is the set of market stress probabilities with Sudden Stop equilibria.

Corollary 1 (Role of Fiscal Capacity)
For any LOLR with fiscal capacity strictly above zero, there is at least one probability of market stress, that with public liquidity provision, shifted from a Sudden Stop to a No Crisis.

Proof As discussed previously, Mature LOLR eliminate sudden stop equilibria. Now, I consider the case for $\bar{\mu}$ strictly between $]0, \mu_A[$. Choose pair $\{\gamma_1^H, 0\}$ and define set $\Omega(\gamma_1^H, 0) = \{z \mid z \leq \omega(\gamma_1^H, 0)\}$. By construction, note that $\omega(\gamma_1^H, 0) = \omega$, thus $\Omega(\gamma_1^H, 0)$ is contained in Ω. Select z equal to ω. Note that $z \epsilon\ \Omega$. Choose any $\bar{\mu}'$ strictly between $]0, \mu_A[$ and define set $\Omega(\gamma_1^H, \bar{\mu}')$ Since $\bar{\mu}' > 0$ and $\omega(\hat{R}, \bar{\mu})$ is strictly decreasing $(\rho_1 > \hat{R})$ with respect to $\bar{u}$ for any $\hat{R}$ including γ_1^H, then $z > \omega(\hat{R}, \bar{\mu}')$ and, thus, z doesn't belong to $\Omega(\gamma_1^H, \bar{\mu}')$ ∎

Corollary 1 states that in economies with an LOLR with some fiscal capacity ($\bar{\mu} > 0$) provision of public liquidity shifts at least one probability from a Sudden Stop in LFE to a No Reserves - No Crisis Equilibrium.

As argued previously, banking entrepreneurs face a trade-off between investment scale and insurance. Since public liquidity provision with some fiscal capacity ($\bar{\mu} > 0$) expands to some degree pledgeable income, it reduces the amount of liquidity that needs to be hoarded at $t = 0$ from $1 - \frac{\rho_0}{\gamma_1^H}$ to $1 - \frac{\rho_0}{(1-\bar{u})\gamma_1^H}$. Thus, in return, banking entrepreneurs are willing to insure against a market stress event with lower probability. This is shown in Figure 5 where the threshold between No reserves equilibria is decreasing with $\bar{\mu}$.

Thus, LOLR intermediation decreases the amount of private liquidity holdings but, at the same time, it increases the set of probabilities of full-scale reinvestment due to self-insurance.

The results in this section also highlight that unintended consequences of ex-post liquidity provision on private self-insurance are contingent on the level of fiscal capacity. When an LOLR can provide liquidity cost-free (mature equilibrium), entrepreneurs do not self-insure, and borrow the maximum amount they can from international markets. In contrast, when an LOLR has underdeveloped fiscal capacity, entrepreneurs start to self-insure for parameter spaces that they didn't self-insure in the LFE. The reason is that LOLR intervention reduces the cost of self-insurance, and, as a result, it becomes more attractive.

Having said that, LOLR with underdeveloped fiscal capacity cannot eliminate sudden stops equilibria for all possible probabilities. In this case, the LOLR has to rely in other instruments to close the gap between pledgeable and total return. One possibility is to accumulate foreign reserves which I analyze below.

3.1 The Role of Reserves

Reserves allow an LOLR to offer funding at a lower marginal cost than international markets ($\bar{R} \leq \gamma_1^H$). Thus, this price effect increases projects pledgeability relative to the laissez faire scenario. Higher pledgeability potentially provides room for more reinvestment.

However, accumulating reserves can end up being wasteful since they are not state-contingent, and, when a boom materializes, an LOLR deviates resources from more productive investments without reaping any of the benefit. Clearly, similar to entrepreneurs, LOLR face a trade-off between insurance and scale.

I derive the optimal behavior of an LOLR in Appendix B.12. I show that, when there is a positive τ at $t = 1$, a LOLR guarantees that bonds issued at international markets are redeemable at $t = 2$ by setting $\hat{R}$ equal to (18). Moreover, without loss of generality, I assume that any stock of reserves that goes unused is rebated to entrepreneurs at the end of $t = 2$.

At $t = 0$, the LOLR faces the trade-off between insurance and scale. Accumulating reserves is not optimal when the economy already has other sources to compensate for moral hazard. Two states of the economy fall within this realm. The first is when the LOLR has sufficient fiscal capacity ($\bar{\mu} \geq \mu_A$) while the second state is when the private sector holds enough liquidity already $x_1^H \geq i(1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H})$. The intuition for this optimal behavior is simple: no need to incur in the opportunity cost when reserves don't provide additional reinvestment.

Corollary 2 (LOLR - Mature Fiscal Capacity Equilibrium)
Whenever $\bar{\mu} \geq \mu_A$, an LOLR would optimally choose F_0 equal to zero in the Mature Fiscal Capacity Equilibrium described in Proposition 3

Proof Choose $\bar{\mu} \geq \mu_A$. Proposition 3 shows that full-scale reinvestment is reached for any x_1^A when $F_0 = 0$. Choosing no stock of reserves, then, generates an expected welfare cost of zero. Suppose that there exists a positive F_0 that creates a lower expected welfare costs. This is not possible since reinvestment cannot be greater than initial investment. Thus, for any $F_0 > 0$, $\psi F_0 \kappa(x_1^H)$ is strictly greater than zero. ∎

Corollary 3 (LOLR - No Crisis Private Hoarding Equilibrium)
*Define set $\Omega(\bar{\mu}, \hat{R}) = \{\ z \mid z \leq \omega(\bar{\mu}, \hat{R})\}$ Whenever $\bar{\mu} < \mu_A$, and $(1-\alpha) \in \Omega^C(\bar{\mu}, \gamma_1^H)$, an LOLR would optimally choose F_0 equal to zero in the **No Crisis Equilibrium - Private Hoarding** described in Proposition 4*

Proof Choose $\bar{\mu} < \mu_A$. Proposition 4 shows accumulating $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$ by entrepreneurs is a best response to F_0 equal to zero when $(1-\alpha) \in \Omega^C(\bar{\mu}, \gamma_1^H)$. Given that $x_1^H = i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]$, would an LOLR choose a F_0 equal to zero? The answer is yes. At this level, entrepreneurs reach full-scale reinvestment during market stress with $\hat{R}$ at γ_1^H, thus it generates an expected welfare cost of zero. Suppose that there exists a positive F_0 that creates a lower expected welfare costs. This is not possible since reinvestment cannot be greater than initial investment. Thus, for any $F_0 > 0$, $\psi F_0 \kappa(x_1^H)$ is strictly greater than zero. ∎

Corollary 2 states that when a Mature LOLR has the possibility to accumulate reserves, it chooses not to. So, in effect, F_0 equal to zero is optimal in a Mature Fiscal Capacity

Equilibrium (Proposition 3).[12] Likewise, if banking entrepreneurs hold sufficient liquidity to reinvest at full-scale as in the **No Crisis Equilibrium - Private Hoarding**, then an LOLR with $\bar{\mu} < \mu_A$ would choose F_0 equal to zero as well (Corollary 3).

What is the case for a LOLR with low fiscal capacity when private liquidity holdings lie strictly between 0 and $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})}\big]$? The actual optimal response depend on the specifics of $L(J^H)$ and parameter values. Yet, still something can be said about the general lines of this behavior. If x_1^H is close enough to $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})}\big]$, the marginal benefit of increasing reinvestment is around zero since already j^H is close to i while the marginal opportunity cost of F_0 is positive. Thus, it is possible that this LOLR accepts some partial liquidation of projects before accumulating reserves. Now, as x_1^H tends to zero, the marginal benefit of higher reinvestment due accumulating reserves increases while its marginal opportunity cost remains constant. Thus, it is possible to reach an interior solution where both entrepreneurs and the LOLR hoard liquidity.

In turn, if x_1^H is zero, the small economy finds itself at the doors of complete shutdown. LOLR intervention, characterized by pair $\{\bar{\mu}, \hat{R}\}$, is attractive for banking entrepreneurs as long as (21) holds.

$$(1-\bar{\mu})\hat{R} \leq \rho_0 \tag{21}$$

Condition 21, by definition, doesn't hold for pairs $\{\bar{\mu}, \gamma_1^H\}$ when $\bar{\mu} < \mu_A$. Thus, in this scenario, an LOLR needs to accumulate reserves to observe some reinvestment. I define function $\bar{F}(\bar{\mu})$ as the minimum amount of reserves such that (21) holds with equality.

$$\bar{F}(\bar{\mu}) = A\frac{\big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)}{1 + \big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)} \tag{22}$$

[12]This result, of course, could be different if an LOLR incurred in some dead weight loss when issuing bonds. However, this dead weight loss has to be sufficiently high in expected terms to compensate for the opportunity cost of accumulating reserves. Moreover, it is plausible to assume that any dead weight loss of issuing bonds is lower in economies with greater than with lower fiscal capacity. Hence, it the qualitative implications of the model would remain the same.

Amount $\bar{F}(\bar{\mu})$ shows that reserves need to compensate for the wedge between liquidity demand and pledgeable income valued with fiscal capacity $(1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H})$. Hence, a lower fiscal capacity implies a greater $\bar{F}(\bar{\mu})$. Likewise, the denominator captures the fact that by reducing banking entrepreneur's disposable endowment at $t = 0$, investment scale is smaller and as such the amount of reserves required is smaller.

Additionally, note that $\bar{F}(\bar{\mu})$ is feasible for any LOLR since it is strictly less than A precisely due to this effect on lower investment scale.

Proposition 5 (LOLR Optimal Response to $x_1^H = 0$)
When $\bar{\mu} < \mu_A$ and x_1^H is zero, define set $\Lambda(\bar{\mu}) = \{z \mid z \leq \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$. The optimal response of a LOLR at $t = 0$ is

$$F_0 = \begin{cases} 0 & \text{if } (1-\alpha) \in \Lambda(\bar{\mu}) \\ \bar{F}(\bar{\mu}) & \text{if } (1-\alpha) \in \Lambda^c(\bar{\mu}) \end{cases}$$

Proof In Appendix B.12

Proposition 5 depicts LOLR's optimal response to $x_1^H = 0$. Just like banking entrepreneurs, LOLR doesn't hoard liquidity in the form of reserves when the probability of a market stress is *relatively low*. This threshold is determined by the ratio between the cost of accumulating the minimum necessary amount of reserves and the welfare losses of a complete shutdown. If $L(0)$ is high enough, then it is possible for set $\Lambda(\bar{\mu})$ to be empty for any feasible $(1 - \alpha)$ (Assumption 3). In such case, an LOLR will always accumulate reserves.

Proposition 6 (No Crisis - Reserves Equilibrium)
For $\bar{\mu} < \mu_A$, $(1-\alpha) \in \Lambda^c(\bar{\mu})$, and Assumptions 1, and 3 hold, in this small open economy

- *Date-0: banking entrepreneurs invest $i = (A - F_0)\kappa(0)$ and do not hoard liquidity ($x_1^L = 0,\ x_1^H = 0$) while LOLR's accumulate $F_0 = \bar{F}(\bar{\mu})$*
- *Date-1: In both states, reinvestment is done at full-scale ($j = i$). In a market stress, entrepreneurs demand i of public liquidity while the LOLR sets $\hat{R}$ equal to $\frac{\rho_0}{1-\bar{\mu}}$ and issues B_1 for a value of $A\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\frac{\kappa(0)}{1+\left[1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\right]\kappa(0)}$*
- *Date-2: Entrepreneurs do not abscond, LOLR collects $\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}i$ and redeems fully $\gamma_1^H B_1$*

Proof The proof consists on showing that at $t = 0$, optimal response functions are consistent. Behavior for $t = 1$ and $t = 2$ follows from decisions on x_1^H and F_0 and can be found in the Appendix. Suppose that F_0 is equal to $\bar{F}(\bar{\mu})$. Therefore, entrepreneurs expect pair $\{\bar{\mu}, \hat{R}\}$ to be equal to ρ_0 which implies an optimal choice of x_1^H equal to zero. Now, suppose that x_1^H is equal to zero. Since the LOLR is $\bar{\mu} < \mu_A$, and $(1-\alpha) \in \Lambda^c(\bar{\mu})$, then it is optimal to choose F_0 equal to $\bar{F}(\bar{\mu})$. ∎

The comparison between the Mature Fiscal Capacity equilibrium and the No Crisis - Reserves Equilibrium (Proposition 6) underscores the main result of this paper. In both equilibria, the LOLR prevent market stress episodes turining into sudden stops, and, reinvestment manages to reach full-scale despite the existence of financial frictions. Moreover, in both equilibria, the private sector doesn't hoard liquidity. The main difference lies in that while a mature LOLR doesn't need to preemptively hoard reserves, the rest of LOLR need to rely in a *sufficient* amount of reserves to provide liquidity ex-post successfully. In other words, a LOLR with a fiscal capacity below μ_A accumulates $\bar{F}(\bar{\mu})$ ex-ante to emulate what a mature LOLR can do ex-post.

An important question is whether LOLR intervention is welfare improving relative to the Laissez Faire Case. Although there are many ways to measure this, I restrict my analysis to whether LOLR eliminates the Sudden Stop episode or not. Not surprisingly, a Mature LOLR eliminates sudden stops for all feasible probabilities of a market stress as in Holmström and Tirole (1998).

Meanwhile, LOLR with lower fiscal capacity eliminates Sudden Stops for $(1-\alpha) \in \Lambda^c(\bar{\mu})$. Hence, for these economies, only probabilities that are part of Ω and part of $\Lambda^c(\bar{\mu})$ shift from a Sudden Stop to a No Crisis equilibrium with LOLR intervention. Once again, the cost of this shift is for LOLR's to accumulate *sufficient* reserves (Corollary 4).

Corollary 4 (LOLR Intervention - Sudden Stop Elimination)
A LOLR intermediation eliminates the Sudden Stop - LFE if

1. $\bar{\mu} \geq \mu_A$ *(Mature LOLR)*
2. $\bar{\mu} < \mu_A$*, when* $(1-\alpha) \in \Omega \cap \Lambda^c(\bar{\mu})$

Proof For any feasible $(1-\alpha)$, the Sudden Stop - LFE exists when $(1-\alpha) \in \Omega$. With $\bar{\mu} \geq \mu_A$, the economy reaches a Mature Fiscal Capacity equilibrium where there is no sudden stop.

While when $\bar{\mu} < \mu_A$, the economy emulates a Mature Fiscal Capacity equilibrium with a No Crisis - Reserves Equilibrium when $(1-\alpha) \epsilon \Lambda^c(\bar{\mu})$. Thus, consequently, LOLR intervention eliminates Sudden Stop for $(1-\alpha)$ that are part of the intersection between Ω and $\Lambda^c(\bar{\mu})$ $\blacksquare$

Given that accumulating reserves is costly, there is the possibility of a Sudden Stop type equilibrium even when a LOLR is present. That is, an equilibrium where the economy fails to borrow from abroad since neither the LOLR nor entrepreneurs chose to accumulate the sufficient amount of liquidity at $t = 0$ to attract funding from foreign lenders during stress event at $t = 1$.

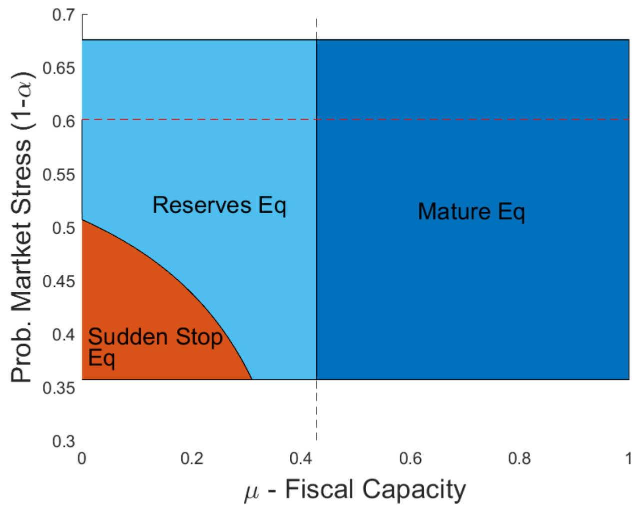
Proposition 7 (Sudden Stop - Reserves Equilibrium)
For $\bar{\mu} < \mu_A$, $(1-\alpha) \epsilon \Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$, and Assumptions 1, and 3 hold, in this small open economy

- *Date-0: banking entrepreneurs invest $i = A\kappa(O)$ and do not hoard liquidity ($x_1^L = 0,\ x_1^H = 0$) while the LOLR doesn't accumulate reserves $\{F_0 = 0\}$*
- *Date-1: Reinvestment only occurs during booms. The LOLR sets $\hat{R}$ equal to γ_1^S. In a market stress, entrepreneurs don't demand public liquidity, and, as a result, the LOLR doesn't issue bonds.*
- *Date-2: Entrepreneurs do not abscond following a boom and pay back foreign lenders. Following a stress event, nothing occurs.*

Proof Suppose that F_0 is equal to zero. Therefore, entrepreneurs expect $\hat{R}$ to be equal to γ_1^H which, together with a $\bar{\mu} < \mu_A$ and $(1-\alpha)\ \epsilon\Omega(\bar{\mu}, \gamma_1^H)$, imply an optimal choice of x_1^H equal to zero. Now, suppose that x_1^H is equal to zero. Since the LOLR is $\bar{\mu} < \mu_A$, and $(1-\alpha) \epsilon \Lambda(\bar{\mu})$, then it is optimal to choose F_0 equal to zero. $\blacksquare$

Figure 6 shows the equilibria with LOLR where banking entrepreneurs don't hold any liquidity for market stress events ($x_1^H = 0$) for feasible pairs $\{1-\alpha, \bar{\mu}\}$. Note that, in contrast to Figure 5, there is an additional area of fiscal capacity, lower than μ_A, where sudden stops are eliminated for all possible values of $(1-\alpha)$. In this numerical example, μ_A is equal to 0.4286, thus, for $\bar{\mu}\ \epsilon \big[0.31, \mu_A\big]$, every feasible $1-\alpha$ belongs to $\Lambda^c(\bar{\mu})$. For a LOLR with fiscal capacity within $\big[0, 0.31\big]$, the LOLR eliminates sudden stops by preemptively accumulating $\bar{F}(\bar{\mu})$ only if the market stress is not sufficiently rare.

Figure 6: Equilibria with LOLR and $x_1^H = 0$



Lastly, a direct consequence of the possible existence of a sudden stop type equilibrium is that some economies in the world economy cannot afford to insure against market stress. Lower fiscal capacity implies they require more reserves. Thus, within the spectre of low fiscal capacities, it is possible to observe an economy with lower fiscal capacity in a Sudden Stop - Reserves Equilibrium while another economy with greater fiscal capacity in a No Crisis -Reserves Equilibrium.

Corollary 5 (Comparative Statics - $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$)
Set $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$ is contracting with respect to $\bar{\mu}$

Proof Choose $\bar{\mu}$ such that $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$ is none empty. Select z equal to the minimum between $\omega(\bar{\mu})$ and $\frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$. Note that z belongs to set $\Lambda(\bar{\mu}) \cap \Omega(\bar{\mu}, \gamma_1^H)$. Select $\bar{\mu}'$ greater than $\bar{\mu}'$. Since both $\omega(\bar{\mu})$ and $\bar{F}(\bar{\mu})$ are strictly decreasing with respect to $\bar{\mu}$, then z doesn't belong to set $\Lambda(\bar{\mu}') \cap \Omega(\bar{\mu}', \gamma_1^H)$ ∎

Corollary 5 shows that set $\Lambda(\bar{\mu})\cap\Omega(\bar{\mu}, \gamma_1^H)$ is contracting with respect to fiscal capacity. Thus, observing an economy with lower fiscal capacity exposed to a sudden stop hinges on whether the LOLR with no fiscal capacity is exposed to sudden stops - the set $\Lambda(0) \cap \Omega(0, \gamma_1^H)$ is non-empty. Whether this set is empty or not, ultimately, depends on how big are the welfare costs perceived by the LOLR in the case of a complete shutdown.

In this section, I have shown the possibility of three different equilibria depending on the level of fiscal capacity and the probability of a market stress. Countries with low fiscal capacity can emulate the ability to provide liquidity ex-post that mature countries have by accumulating reserves. However, low fiscal capacity itself can deter countries from choosing to emulate.

3.2 Multiple Equilibria

Figure 7 presents the complete set of equilibria identified for this numerical example. Note that when a market stress event is not *rare*, there is multiple equilibria in environments with a LOLR whose fiscal capacity is below the maturity threshold: one with private liquidity hoarding, and the other with public liquidity hoarding through the accumulation of reserves.

Proposition 8 (Multiple Equilibria)
As long as $\bar{\mu} < \mu_A$*, Assumptions 1 and 3 hold, and* $(1-\alpha)\ \epsilon\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$*, then, at least two equilibria co-exist:*

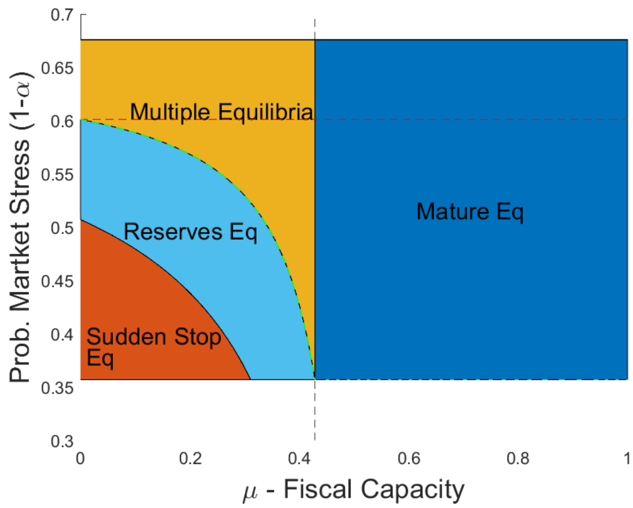
- *No Crisis - Private Hoarding Equilibrium*
- *No Crisis - Reserves Equilibrium*

Proof Note that I assume that $\bar{\mu} < \mu_A$, and Assumptions 1 and 3 hold. By Definition, any $(1-\alpha)$ that belongs to set $\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$ is part of $\Omega^C(\bar{\mu}, \gamma_1^H)$. Thus, No Crisis - Private Hoarding Equilibrium exists according to Proposition 6 and Corollary 3. Likewise, any $(1-\alpha)$ that belongs to set $\Lambda^C(\bar{\mu}) \cap \Omega^C(\bar{\mu}, \gamma_1^H)$ is part of $\Lambda^C(\bar{\mu})$. Thus, No Crisis - Reserves Equilibrium exists according to Proposition 6. Now, I show that this intersection is non-empty. First, suppose that $\omega(\bar{\mu}, \gamma_1^H) \geq \frac{\psi\kappa(0)}{L(0)}$. Choose $z\ \epsilon\Omega^C(\bar{\mu}, \gamma_1^H)$ then $z\ \epsilon\Lambda^C(\bar{\mu})$. Suppose that $\omega(\bar{\mu}, \gamma_1^H) < \frac{\psi\kappa(0)}{L(0)}$. Choose $z\ \epsilon\Lambda^C(\bar{\mu})$ then $z\ \epsilon\Omega^C(\bar{\mu}, \gamma_1^H)$ ∎

Yet, multiple equilibria is not a feature of environments under Mature LOLR. As argued by Farhi and Tirole (2012), multiple equilibria occurs in these type of models because strategic complementarities appear between entrepreneurs self-insurance choices due to a costly untargeted policy instrument. In this setting, liquidity provision policies are only costly for LOLR that require foreign reserves to implement them.

To see this, consider if, under a LOLR with $\bar{\mu} < \mu_A$, an entrepreneur would benefit from hoarding liquidity when the rest of entrepreneurs do not. The answer is no, and the

Figure 7: Equilibria with LOLR



reason is that, as long as the probability of the market stress is high enough, the LOLR would optimally choose to accumulate reserves which allows it to implement a $\hat{R}$ sufficiently low that every entrepreneurs reaches full-scale reinvestment, even with no market liquidity. Similarly, would an entrepreneur benefit from choosing $x_1^H = 0$ if all others choose to hoard liquidity? Again, the answer is no. This time, the LOLR would not accumulate reserves, and, therefore, it would not be able to provide an LLP with $\hat{R}$ lower than γ_1^H forcing the deviating entrepreneur to shutdown while others would reinvest at full-scale using their market liquidity.

In contrast, a Mature LOLR allows for full-scale reinvestment without the need to reduce the cost of liquidity. Thus, regardless of what other entrepreneurs do, an entrepreneur can always ask for a transfer at date-1 if necessary, and the Mature LOLR has the capacity to provide it.

The coexistence of this two equilibria underscores another important feature of this model. There are, under the environment with a low fiscal capacity LOLR, two ways to circumvent moral hazard: private liquidity hoarding or accumulation of reserves.

When the probability of a stress event is relatively high, both entrepreneurs and the LOLR are willing to hoard liquidity ex-ante. However, it is not optimal for either to hold liquidity if it expects the other to do the hoarding. This shows that private and public hoarding are substitute instruments to solve the same problem.

4 Tax Revenue and Crowding Out Mechanisms

In this section, I discuss how low fiscal capacity impairs an LOLR's ability to increase aggregate liquidity during market stress episodes.

During a market stress, Equation (23) establishes $\bar{j}$ as the maximum amount of aggregate liquidity that this domestic economy can supply. This upper bound is equal to the sum of aggregate market and aggregate funding liquidity available at $t = 1$. That is, private liquidity hoarding (x_1^H), the stock of reserves valued at F_0, and parameters $\bar{\phi_1 j}$ and $\bar{B}_1$ which denote the maximum amount that can be borrowed by entrepreneurs and the LOLR, respectively, from international markets.

$$\bar{j} \equiv (x_1^H + F_0) + \bar{\phi_1 j} + \bar{B}_1 \tag{23}$$

An economy can survive intact a market stress episode as long as the maximum supply of liquidity, $\bar{j}$, is equal or greater than i, which is the amount required for full-scale reinvestment. Since x_1^H and F_0 were chosen before the time the aggregate shock is realized at $t = 1$, the outcome of a market stress episode depends on what lies behind $\bar{\phi_1 j} + \bar{B}_1$.

Proposition 9 (Upper Bound on Aggregate External Liabilities)
Define function $\bar{T}(\mu) : [0, 1] \rightarrow [0, 1]$ as the aggregate tax rate of the economy where

$$\bar{T}(\mu) \equiv min\{\frac{1-\theta}{1-\bar{\mu}}, 1\}$$

Then, the equilibrium upper bounds on public liquidity and on aggregate external liabilities, respectively, during a market stress are given by

$$\bar{T}(\mu)\frac{\rho_1}{\gamma_1^H}j = \bar{B}_1$$

$$\bar{\phi_1 j} + \bar{B}_1 = min\{\frac{\rho_0}{\gamma_1^H}j + \bar{\mu}\bar{T}(\mu)\frac{\rho_1}{\gamma_1^H}j, \frac{\rho_1}{\gamma_1^H}j\}$$

Proof In the Appendix. ▮

Proposition 9 establishes the upper bound on external liabilities in any equilibrium as a function of the government's fiscal capacity. With no government intervention, $\bar{\phi_1}$J cannot be greater than pledgeable output $\frac{\rho_0}{\gamma_1^H}j$ in a default free equilibrium (Constraint 14). The bound of $\frac{\rho_1}{\gamma_1^H}j$ exists to guarantee that the economy is solvent in equilibrium. The level of fiscal capacity moves the effective upper bound on external liabilities between $\frac{\rho_0}{\gamma_1^H}j$ and $\frac{\rho_1}{\gamma_1^H}j$.

Specifically, there are two channels,which are captured by the term $\bar{\mu}\bar{T}(\mu)$, through which low fiscal capacity impairs the aggregate amount of liquidity that the economy can produce.

First, as in Bocola and Lorenzoni (2020), Benigno and Robatto (2019), and Farhi and Maggiori (2018), low fiscal capacity reduces the aggregate tax rate $\bar{T}(\mu)$ that a government can impose on its economy, and, as a result, it limits the amount of backed-up public liquidity that can be issued. Note that $\bar{T}(\mu)$ falls to less than 1 for $\bar{\mu} < \theta$, and reaches a minimum level of $1-\theta$ when $\bar{\mu}$ is zero. That is, the government can credibly collect from its economy the same share of total output as foreign lenders when it has no fiscal capacity. I refer to this effect as the tax rate channel.

Proposition 9 also establishes that increases in total tax revenue $\bar{T}(\mu)\frac{\rho_1}{\gamma_1^H}$ do not expand the upper bound on external liabilities on a one to one basis but, instead, on a one to $\bar{\mu}$ basis. Since $\bar{\mu}$ is utmost 1, some loss of liquidity occurs when public liquidity intervenes. I refer to this effect as the crowding out channel.

To see the crowding out channel more explicitly, I rewrite $\bar{\phi_1}j$ as a function of market liquidity and public liquidity by substituting (23) in Proposition 9. Equation 24 shows how the upper bound on private funding liquidity changes with x_1^A, F_0 and B_1.

$$\bar{\phi}_1 j = \left[\frac{\rho_0}{\gamma_1^H}\left[x_1^H + F_0\right] + \bar{B}_1\left[\bar{\mu} - \left(1 - \frac{\rho_0}{\gamma_1^H}\right)\right]\right]\frac{1}{1-\frac{\rho_0}{\gamma_1^H}} \tag{24}$$

When the economy hoards more liquidity, either through private agents (x_1^H) or the LOLR in the form of reserves (F_0), it has more resources that it can use to reinvest during market stress episodes. By doing so, it increases the size of pledgeable output which, in turn, is used to attract more resources through private borrowing.

The effect of public liquidity on private liquidity depends on the level of fiscal capacity. As argued by Holmström and Tirole (2011), LOLR's brokerage role potentially increases aggregate pledgeable income that the economy can offer to foreign investors. To be more precise, a marginal increase in B_1 produces an additional $\frac{\rho_0}{\gamma_1^H}$ of pledgeable output. However, private borrowing is backed up by pledgeable income, net of the taxes that sustain some share of public borrowing. Therefore, the net change in aggregate pledgeable output is equal to

$$\frac{\partial \bar{\phi_1} j}{\partial \bar{B}_1} = \frac{\bar{\mu}}{1 - \frac{\rho_0}{\gamma_1^H}} - 1$$

In equilibrium, what effect is bigger? As fiscal capacity increases, the brokerage effect strengthens while the crowding out effect weakens. This occurs because greater fiscal capacity allows the LOLR to back up the additional public borrowing with *new* pledgeable output. That is, pledgeable output that is not backing up private liquidity.

When $\bar{\mu}$ is greater or equal to μ_A, greater public liquidity provision *crowds in* $\bar{\phi_1} j$. For economies with mature fiscal capacity, B_1 and $\phi_1 j$ are complements. In contrast, when fiscal capacity is underdeveloped, B_1 and $\phi_1 j$ are substitute assets, and increases in public liquidity *crowd out* private liquidity.

How important is the crowding out channel relative to the tax rate channel? To see this, I break down the fiscal capacity parameter $\bar{\mu}$ into the crowding out channel, denoted by μ_{CO}, and the parameter μ_T that denotes the tax rate channel. I rewrite the upper bound of external aggregate liabilities as

$$min\{\frac{\rho_0}{\gamma_1^H} j + \mu_{CO} \bar{T}(\mu_T) \frac{\rho_1}{\gamma_1^H} j,\ \frac{\rho_1}{\gamma_1^H} j\}$$

Proposition 10 establishes the thresholds for a mature fiscal capacity in a version of the model where only one of the two channels is active. When only the tax rate is on ($\mu_{CO} = 1$), economies with $\bar{\mu} \geq \bar{\mu}_T$ are mature, whereas when only the crowding out channel is turned on ($\mu_T = 1$), economies with $\bar{\mu} \geq \bar{\mu}_{CO}$ are mature.

Proposition 10 (Thresholds of Mature Fiscal Capacity)
Let Assumptions 1 and 3 hold. Define thresholds of fiscal capacity

$$\bar{\mu}_T \equiv \frac{1 - 2\frac{\rho_0}{\gamma_1^H}}{1 - \frac{\rho_0}{\gamma_1^H}} \qquad \bar{\mu}_{CO} \equiv \frac{\gamma_1^H}{\rho_1}[1 - \frac{\rho_0}{\gamma_1^H}]$$

Consider an LOLR with fiscal capacities $\{\mu_T, \mu_{CO}\}$. Then, full-scale reinvestments are possible, even with $x_1^H + F_0$ equal to zero, when

- $\mu_{CO} = 1$ *and* $\mu_T \geq \bar{\mu}_T$
- $\mu_T = 1$ *and* $\mu_{CO} \geq \bar{\mu}_{CO}$

Proof In the Appendix. ∎

I previously showed that economies with mature LOLR eliminate sudden stops in equilibrium without preemptive liquidity hoarding. Therefore, only economies with fiscal capacity levels less than the mature thresholds potentially demand reserves.

Corollary 6 states that, for any set of feasible parameters, there is a ranking of the maturity thresholds with μ_A always sitting at the top, above $\bar{\mu}_{CO}$ and $\bar{\mu}_T$.

In other words, when the two channels are turned off, an economy needs greater investments to reach a level of fiscal capacity that guarantees efficient liquidity provision abilities than when only one level mechanism is active. This implies that the crowding out and the tax revenue effects do not offset each other, and, consequently, produce a greater set of underdeveloped fiscal capacities.

Corollary 6 (Order of thresholds for mature fiscal capacity)
Let Assumptions 1 and 3 hold. Define

$$\gamma^- = \rho_0 + \frac{\rho_1}{2} - \rho_1\sqrt{\theta - 3/4}; \qquad \gamma^+ = \rho_0 + \frac{\rho_1}{2} - \rho_1\sqrt{\theta - 3/4}$$

For a set of a feasible parameters of the model

- *If* $\theta > 3/4$ *and* γ_1^H *between max*$\{\gamma^-, 2\rho_0\}$ *and* γ^+

$$\mu_A > \bar{\mu}_T > \bar{\mu}_{CO} > 0$$

- *Otherwise*

$$\mu_A > \bar{\mu}_{CO} > max\{\bar{\mu}_T, 0\}$$

Proof In the Appendix ∎

How does the crowding out channel compare to the tax revenue effect? Corollary 6 argues that the order is parameter dependent.

To see this, note that, as long as the assumptions of the model are satisfied, $\bar{\mu}_{CO}$ is always positive, while $\bar{\mu}_T$ is only positive when the funding cost of a market stress is sufficiently large relative to the economy's pledgeable output ($\gamma_1^H > 2\rho_0$). Therefore, in contrast to the tax revenue channel, an active crowding out channel always produces levels of fiscal capacity that require liquidity hoarding for successful liquidity provision programs.

Figure 8 presents μ_A, $\bar{\mu}_{CO}$, and $\bar{\mu}_T$ as a function of feasible values of γ_1^H. Note that, for this specific numerical example, $\bar{\mu}_{CO}$ is positive and greater than $\bar{\mu}_T$ for all possible γ_1^H while $\bar{\mu}_T$ becomes positive only when market stress funding costs are above 1.6. Thus, the set of equilibria presented as illustration in this paper is driven mainly by the crowding out channel.

Moreover, Corollary 6 also indicates that $\bar{\mu}_T$ *can* be greater than $\bar{\mu}_{CO}$ only in economies subject to strong moral hazard ($\theta > 3/4$). The reason is that the smaller is pledgeable output, the government needs more tax revenue to issue enough public debt to reach full-scale reinvestment, while, simultaneously, there is less pledgeable output to crowd out.

Overall, one can say that the lack of fiscal capacity impairs government liquidity provision generally through the crowding out channel, and not by limiting the amount of resources that a government can collect. And, even in the limited cases where the tax revenue channel is stronger, the crowding out effect is always present.

This previous discussion underscores that public liquidity programs are successful as long as they are able to provide stores of value backed by *new* pledgeable output. Thus, in contrast to Bocola and Lorenzoni (2020), the driving force behind fiscal capacity in my framework is not a matter about the amount of resources that a government has available

Figure 8: Mature and Underdeveloped Fiscal Capacity Sets - μ_A, $\bar{\mu}_{CO}$ and $\bar{\mu}_T$

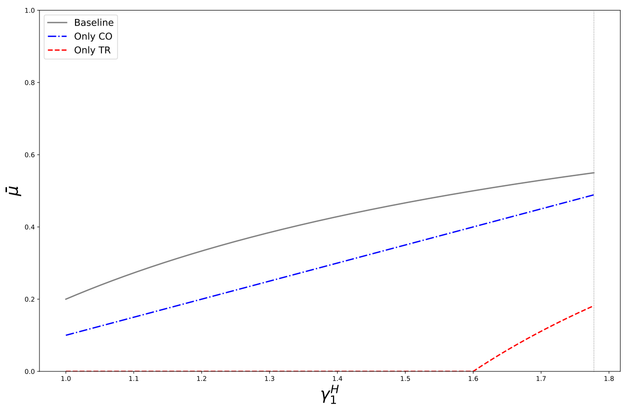


Figure 8 presents three lines that depict μ_A (solid gray), $\bar{\mu}_{CO}$ (dash-dot blue), and $\bar{\mu}_T$ (dash red) as functions of feasible values of the funding cost in the high state. Each line separates the space of fiscal capacity (y axis) into two groups. Above the line consists of the values of fiscal capacity considered mature while below each line comprises the values for underdeveloped LOLRs. The vertical line reflects the upper bound on γ_1^H in Assumption 3.

to back public debt, but the sources of those funds.

5 Fiscal Capacity and Foreign Reserves in the data

In this section, I empirically assess four results of the theoretical model. To do so, I construct an unbalanced panel of 44 countries between 1991 and 2019, of which 14 are advanced economies while the rest are emerging and low income countries according to the IMF. A full description of the data and its sources can be found in Section A.1 of the Empirical Appendix.

The empirical exercises in this section are cross country by design. This comes from the fact that the model takes as given a level of fiscal capacity, and abstracts from any potential dynamics coming from fiscal capacity building. Different levels of fiscal capacity determine different equilibria. Thus, the appropriate interpretation of the model in the data is a cross-country comparison, and not a within country exercise.

Fiscal capacity is an unobservable feature of a government. As a proxy, Besley et al. (2013) use the share of total taxation collected through income tax. These authors argue that how a government collects taxes reveals the investments it has previously made to develop its ability to collect taxes.[13] Thus, collecting a greater share of tax revenue through more sophisticated taxes signals a government that has made investments to develop its fiscal capacity.

Additionally, the theoretical model and its results emphasize the importance of an LOLR's ability to collect revenue directly from individuals, and not from projects. Therefore, instead of using total revenue from income tax as Besley et al. (2013), I use a more stringent proxy for fiscal capacity: revenue from income taxes levied on individuals. For a given level of total taxation, I interpret a greater share of tax revenue coming from income tax levied to people as an economy with greater fiscal capacity.[14]

[13] "We view the creation of fiscal capacity as a product of investments in state structures—including monitoring, administration and compliance through e.g.,well-trained tax inspectors and an efficient revenue service" (Besley et al., 2013, p.52)

[14] In Section A.4 of the Empirical Appendix, I present all the exercises from this section using Besley and Persson (2013) proxy: Total income tax revenue as percentage of tax revenue from the World Development Indicators dataset. With this other proxy for fiscal capacity, the sample of countries increases to 73 and, overall, the main results remain robust.

The main exercise of this section builds on the extensive literature that empirically estimates different motives behind foreign reserves accumulation, such as Aizenman and Lee (2007), Obstfeld et al. (2010), and Ghosh et al. (2017). The predominant econometric specification in this literature assumes that the stock of foreign reserves, usually normalized by the respective Gross Domestic Product, in country j at time t ($y_{j,t}$) is a linear function of a set of variables ($Z_{j,t-1}$) that proxy different potential motives.

$$ln(y_{j,t}) = \beta_0 + \beta_1 Z_{j,t-1} + \beta_3 FC_{j,t-1} + \alpha_t + \varepsilon_{j,t} \tag{25}$$

I follow Obstfeld et al. (2010) by breaking down the right hand side of Equation 25 into different explanatory sets of variables behind reserves accumulation: the traditional set, the financial stability set, and the mercantalist set. The traditional and the financial stability sets capture shocks to a country's current and financial accounts, respectively. Both of these sets reflect precautionary motives behind foreign reserves accumulation. In contrast, the mercantalist set interprets the build up in foreign reserves as a by product of governments intent to devalue its exchange rate. A full description of the variables in each set can be found in Section A.2 of the Empirical Appendix.

I extend the predominant econometric specification by including a *Fiscal Capacity Set*, denoted by $FC_{j,t-1}$. The variables in this set are chosen based on the theoretical model of the paper. The core variables describe the tax structure of the economy: i) revenue from income tax to people as proxy for fiscal capacity, ii) the share of total tax revenue that is collected via corporate income tax, and iii) total tax revenue as a percentage of GDP to control for the scale of total taxation.

As discussed above and in contrast to Besley et al. (2013), I break down income tax revenue between individuals and businesses, and I use only the former as proxy for fiscal capacity. Nevertheless, I also include corporate income tax revenue in the regression. The reason is that, in my model, taxes levied on projects reduce their pledgeable output, and, as a result, increase the likelihood of demanding foreign reserves in equilibrium. I expect a positive correlation between corporate income taxes and foreign reserves.

Additional to tax variables, Equation (22) indicates that the stock of reserves is a function of an economy's pledgeable income which I proxy with the size of existing private and public external liabilities, as % of GDP. This selection follows Dominguez (2009) who use foreign liabilities as a measure of financial development, plus, I separate between private and public liabilities because, in the model, ρ_0 refers exclusive to private pledgeable income. I also include in the set an economy's private liquid asset position (variable x_1^H in the model) that I proxy with each country's international investment net position in portfolio debt securities. This choice assumes that the public sector only accumulates liquid assets in the form of reserves. Lastly, I include year and income group fixed effects to control for international funding costs (γ_t) and their probability of occurrence ($\alpha, 1-\alpha$) which are assumed, in accordance to the Global Financial Cycle hypothesis, to be time varying but common between countries and income groups.

The most relevant empirical implication of the theoretical model is that insufficient fiscal capacity is a potential motive behind foreign reserves accumulation. Specifically, a positive demand for reserves in equilibrium is only possible when $\bar{\mu}$ is less than μ_A. This implies that in the data, when comparing two countries with different levels of fiscal capacity, the likelihood of observing foreign reserves accumulation for liquidity purposes is lower for the country with higher fiscal capacity. I summarize this first empirical implication with Remark 1.

Remark 1. Negative elasticity between fiscal capacity and foreign reserves

Table 1 presents the results of estimating Equation 25 where columns (1), (3), and (5) exhibit the estimates when only including the Fiscal Capacity set in the regression, while columns (2), (4), and (6) include the traditional, financial stability, and mercantalist sets of variables as well.

Consistent with Remark 1, I find an elasticity of -0.34 between foreign reserves and income tax on individuals for the period of analysis. This estimate falls, in absolute value, to -0.18 but remains statistically significant after including other motives behind foreign reserves accumulation. The elasticity estimates for fiscal capacity are negative both for the years between the East Asian crisis up to the Global Financial Crisis (Columns 3 and 4), as well as for the years that followed (Columns 5 and 6). However, the estimated elasticity of fiscal capacity in Column (6) losses statistical significance. This can be explained by shifting motives behind foreign reserves accumulation, in line with Ghosh et al. (2017). While fiscal capacity was a relevant motive during the strong worldwide build-up of reserves before the GFC, other variables different than fiscal capacity have more statistical power to explain cross-country differences in recent years.

I find a robust positive elasticity between corporate income tax revenue and foreign reserves (Table 1). This result is in line with Remark 1 and the model, given that greater tax burden on projects, reduces the net private pledgeable output, and countries are more likely to demand reserves.

Table 1: Foreign Reserves and Fiscal Capacity

	1991-2019		Pre-GFC		Post-GFC	
	(1)	(2)	(3)	(4)	(5)	(6)
Income Tax People (% TR, log)	-0.34***	-0.18**	-0.36***	-0.18*	-0.31**	-0.05
	(0.11)	(0.08)	(0.13)	(0.09)	(0.13)	(0.07)
Income Tax Business (% TR, log)	0.46*	0.47***	0.61**	0.58**	0.39	0.41**
	(0.23)	(0.14)	(0.23)	(0.22)	(0.26)	(0.16)
Tax Revenue (% GDP, log)	0.72*	0.85**	0.57	0.81*	0.84*	0.87**
	(0.40)	(0.33)	(0.42)	(0.47)	(0.48)	(0.35)
Private Foreign Liabilities (% GDP, log)	0.06	-0.08	-0.02	0.13	0.14	-0.32**
	(0.12)	(0.11)	(0.13)	(0.18)	(0.15)	(0.12)
Public Foreign Liabilities (% GDP, log)	0.17**	0.12*	0.20**	0.16	0.19**	0.08
	(0.07)	(0.07)	(0.08)	(0.11)	(0.09)	(0.07)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	826	747	289	258	425	393
R^2	0.42	0.66	0.39	0.61	0.41	0.76
Countries	44	43	37	37	44	41

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is log units of foreign reserves, excluding gold, as % of GDP. Standard errors in parenthesis clustered by country. Time and income group fixed effects are not reported but are included in every regression. Every independent variable is lagged one year. Pre-GFC consists of years 1998-2007 and Post-GFC consists of years 2010-2019. Complete regression results in Table A.4 in the appendix.

The complete regression results behind Table 1 are available in Table A.4 of the Appendix. I find that imports, broad money, the exchange rate regime, and the level of development of the economy proxied with GDP per capita are also correlated with the level of foreign reserves, along side fiscal capacity. I do not, however, find any evidence for the mercantalist motive as an explanatory set behind foreign reserves accumulation. Lastly, Table A.10 in the Appendix that the empirical support for Remark 1 is robust to using total income tax revenue as proxy for fiscal capacity.

Overall, Table 1 no only provides empirical support to Remark 1 but it also sets up the lack of fiscal capacity as an additional motive behind foreign reserves accumulation beyond those currently considered in the literature.

Remark 2 argues that the elasticity between reserves and fiscal capacity is contingent on the level of fiscal capacity. In equilibrium, accumulating reserves is not necessarily optimal for economies with underdeveloped fiscal capacity, specially for those with minimal levels of fiscal capacity. This occurs because the lower fiscal capacity is, the greater is the the cost of accumulating reserves. Thus, an LOLR with low fiscal capacity might find it too expensive to hoard liquidity relative to the expected cost of a liquidity crisis. In the data, one should observe a stronger cross-country negative elasticity between foreign reserves and fiscal capacity as countries from the left tail of the fiscal capacity distribution are excluded from the sample (Remark 2).

Remark 2. *The negative cross-country correlation between foreign reserves holdings and fiscal capacity should increase, in absolute value, as countries with low fiscal capacity are excluded from the sample*

To test Remark 2, I divide the sample into percentiles according to their fiscal capacity. Table 2 presents the results of running Equation 25 for different subsamples. Column (1) presents the estimates for the whole sample, while Column (2) show the estimates after excluding the first 10 percentiles, Column (3) the first 20 percentiles, Column (4) the first 30 percentiles, Column (5) the first 40 percentiles, and, finally, Column (6) whose estimates correspond only to the observations in the upper half of the sample. Colummns (7) to (12) do the same but for regressions including the traditional, financial stability, and mercantalist sets as well. Full regression results are available in the Appendix.

Table 2: Foreign Reserves and Fiscal Capacity by Percentiles

	Fiscal Capacity Set						+ Other Sets					
	(1) Full Sample	(2) ≥p10	(3) ≥p20	(4) ≥p30	(5) ≥p40	(6) ≥p50	(7) Full Sample	(8) ≥p10	(9) ≥p20	(10) ≥p30	(11) ≥p40	(12) ≥p50
Income Tax People (% TR, log)	-0.34***	-0.56***	-0.80***	-1.07***	-1.25***	-1.32**	-0.18**	-0.32**	-0.33**	-0.48*	-0.46	-0.56
	(0.11)	(0.19)	(0.24)	(0.29)	(0.40)	(0.50)	(0.08)	(0.13)	(0.16)	(0.25)	(0.35)	(0.41)
Income Tax Business (% TR, log)	0.46*	0.42*	0.44*	0.41*	0.40	0.39	0.47***	0.47***	0.46***	0.43***	0.46***	0.41**
	(0.23)	(0.23)	(0.23)	(0.23)	(0.27)	(0.37)	(0.14)	(0.14)	(0.15)	(0.14)	(0.13)	(0.16)
Tax Revenue (% GDP, log)	0.72*	0.78*	0.97*	1.18*	1.17*	1.33	0.85**	0.90**	0.80**	0.79**	0.87***	1.03***
	(0.40)	(0.44)	(0.51)	(0.59)	(0.66)	(0.85)	(0.33)	(0.34)	(0.31)	(0.31)	(0.32)	(0.37)
Private Foreign Liabilities (% GDP, log)	0.06	0.05	0.03	0.03	0.04	0.05	-0.08	-0.13	-0.16	-0.20*	-0.19	-0.11
	(0.12)	(0.13)	(0.14)	(0.16)	(0.17)	(0.20)	(0.11)	(0.11)	(0.11)	(0.12)	(0.12)	(0.15)
Public Foreign Liabilities (% GDP, log)	0.17**	0.20***	0.18**	0.14	0.11	0.10	0.12*	0.13*	0.12	0.05	0.01	-0.02
	(0.07)	(0.07)	(0.08)	(0.10)	(0.12)	(0.13)	(0.07)	(0.07)	(0.08)	(0.10)	(0.10)	(0.12)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	826	749	666	584	501	418	747	673	590	513	441	366
R^2	0.42	0.45	0.51	0.59	0.60	0.57	0.66	0.68	0.72	0.77	0.78	0.78
Countries	44	41	39	36	31	28	43	40	38	35	29	27

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is log units of foreign reserves, excluding gold, as % of GDP. Standard errors in parenthesis clustered by country. Time and income group fixed effects are not reported but are included in every regression. Every independent variable is lagged one year. Columns with $\geq pXX$ refer to estimates from a regression that includes observations above XX percentile of the fiscal capacity distribution. Complete regression results in Table A.5 of the appendix.

Results from Table 2 support Remark 2. As we exclude from the estimation the observations on the left hand side of the fiscal capacity distribution, the negative elasticity between reserves and the income tax revenue levied to individuals becomes larger (in absolute terms). This is true regardless other motives besides the fiscal capacity set are included in the regression.

A third result in the theoretical model is the possibility of multiple equilibria. Economies can insure themselves either through foreign reserves accumulation or with private liquidity hoarding. This suggests that, for liquidity purposes, private liquidity assets and foreign reserves are substitutes. However, the model suggests that this is only true for countries with underdeveloped fiscal capacity. Remark 3 leverages on this heterogeneity to test for the existence of multiplicity in the data.

Remark 3. *Foreign reserves holdings and private liquid assets are more likely to be substitute assets as countries have lower fiscal capacity*

To test Remark 3 specifically, I run again Equation 25 but this time I include fiscal capacity quartile dummies, and interactions of these dummies with the variable Portfolio Debt Net IIP. I leave out the dummy and the interaction of the lowest quartile, thus, results are all relative to observations with the lowest fiscal capacity. Table 3 presents selected estimates of this regression, where column (1) includes only the fiscal capacity set while column (2) includes also as controls the traditional, financial stability and mercantalist sets. Complete regressions results are, once again, available in the empirical appendix (Table A.6).

Table 3: Foreign Reserves and Private Liquid Assets

	(1)	(2)
Portfolio Debt Net IIP (% GDP)	-0.001	-0.002
	(0.003)	(0.002)
inter. with Quartile 2 FC Dummy	0.012***	0.013***
	(0.004)	(0.003)
inter. with Quartile 3 FC Dummy	0.008*	0.011***
	(0.004)	(0.003)
inter. with Quartile 4 FC Dummy	0.015***	0.012***
	(0.004)	(0.002)
Income Tax People (% TR, log)	-0.219***	-0.104
	(0.077)	(0.071)
Quartile 2 FC (dummy)	0.384***	0.172
	(0.111)	(0.148)
Quartile 3 FC (dummy)	0.357*	0.162
	(0.188)	(0.201)
Quartile 4 FC (dummy)	-0.483**	-0.520**
	(0.209)	(0.195)
Income Tax Business (% TR, log)	0.431**	0.396***
	(0.213)	(0.139)
Tax Revenue (% GDP, log)	0.908**	0.978***
	(0.417)	(0.309)
Private Foreign Liabilities (% GDP, log)	0.117	-0.135
	(0.121)	(0.090)
Public Foreign Liabilities (% GDP, log)	0.069	0.037
	(0.063)	(0.059)
Observations	826	747
R^2	0.57	0.71
Countries	44	43

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is log units of foreign reserves, excluding gold, as % of GDP. Standard errors in parenthesis clustered by country. Time and income group fixed effects are not reported but are included in every regression. Every independent variable is lagged one year. Complete regressions are available in Table A.6 of the appendix.

The estimates in Columns (1) and (2) of Table 3 indicate a negative, although not significant, correlation between foreign reserves and private liquid assets for observations in the bottom quartile of the fiscal capacity distribution. The estimates of the interactions suggest that the same correlation in the second, third and fourth quartiles are statistically greater relative to the first quartile, linear combinations of these estimates indicate positive and significant correlations for each of these groups.

Results in Table 3 provide partial evidence in favor of Remark 3. The negative correlation between reserves and private assets for observations in the left tail of the fiscal capacity distribution suggest that these assets are substitutes, while they are complements in the rest of the fiscal capacity distribution. Yet, the negative estimate is not significant. Thus, these exercises indicate that if multiplicity is present in the data, then it is present only for countries with minimal levels of fiscal capacity.

Remark 4 establishes the empirical implication of the fourth theoretical result. I show in the model that the lack of fiscal capacity impairs public liquidity provision through two channels: by limiting tax revenue and by crowding out private liquidity. If the second channel exists, then one should observe that the correlation between private and public external liabilities is contingent on fiscal capacity.

Remark 4. *Private and public external liabilities are less likely to be substitutes and more likely to be complements as fiscal capacity increases*

I run Equation 26 to test Remark 4. This specification is the empirical counterpart of Equation 24. Hence, I have the logarithm of private foreign liabilities as dependent variable ($pr_{j,t}$) as a function of contemporaneous public foreign liabilities in logs ($pu_{j,t}$), dummies of the second, third and fourth fiscal capacity quartiles ($d^q_{j,t-1}$), interactions of these dummies with $pr_{j,t}$, and a matrix of controls $X_{j,t-1}$.

In $X_{j,t-1}$, I included the sum of reserves and portfolio net position debt as a proxy for $x^H_1 + F_0$, the proxy of fiscal capacity, time and income group fixed effects, as well as the sets of controls in the traditional and the financial stability models.[15]

[15] I do not include the mercantalist set since the purpose of these variables is to capture a government's

$$ln(pr_{j,t}) = \delta ln(pu_{j,t}) + \Gamma X_{j,t-1} + \sum_{q=2}^{4} \lambda_q d^q_{j,t-1} + \sum_{q=2}^{4} \delta_q d^q_{j,t-1} \cdot ln(pu_{j,t}) + \eta_{j,t} \quad (26)$$

Strong evidence in favor of Remark 4 would be a negative estimate for δ paired with a positive $\delta + \delta_4$, and the estimates of $\delta + \delta_2$ and $\delta + \delta_3$ in between.

Table 4: Crowding Out Channel and Fiscal Capacity

	(1)	(2)	(3)	(4)
Public Foreign Liabilities (% GDP, log)	0.41***	0.13***	0.14*	-0.03
	(0.08)	(0.04)	(0.08)	(0.04)
inter. with Quartile 2 FC Dummy			0.31	0.03
			(0.19)	(0.09)
inter. with Quartile 3 FC Dummy			0.37***	0.22***
			(0.13)	(0.06)
inter. with Quartile 4 FC Dummy			0.53***	0.36**
			(0.18)	(0.14)
Income Tax People (% TR, log)	-0.06	-0.05	0.00	0.08*
	(0.10)	(0.04)	(0.09)	(0.04)
Quartile 2 FC (dummy)			1.46	0.17
			(0.88)	(0.36)
Quartile 3 FC (dummy)			1.27**	0.66**
			(0.49)	(0.27)
Quartile 4 FC (dummy)			2.17***	1.04**
			(0.60)	(0.50)
Reserves + Portfolio Debt Net IIP (% GDP)	0.00*	-0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Observations	825	789	825	789
R^2	0.41	0.83	0.47	0.85
Countries	44	43	44	43

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is log units of private foreign liabilities (% of GDP) Standard errors in parenthesis clustered by country. Time and income group fixed effects are not reported but are included in every regression. Every independent variable is lagged one year. Complete regression results available in Table A.7 of the appendix.

Table 4 presents the results of running Equation 26. Column (1) shows a positive estimate for δ which falls in half (Column 2) when including the the variables from the traditional and financial stability sets. This result indicates that, on average in our sample, there appears to be crowding in effect from public liabilities to private external liabilities.

However, once we include the interactions between public external borrowing and fiscal capacity quartiles, we find that the correlation between private and public foreign liabilities is heterogeneous. Both Columns (3) and (4), where the difference is that the latter includes the traditional and financial stability sets, show that economies at the top half of the fiscal

intentions to devalue the exchange rate, which shouldn't have a direct relationship with private decisions on external borrowing.

capacity distribution have statistically greater and positive correlations than the bottom half. Moreover, Column (4) shows that economies in the first and second quartile have a negative, albeit not significant, correlation between private and public external borrowing.

Table 4 presents partial evidence in support of Remark 4. While I find a heterogeneous relationship between private and external liabilities across the fiscal capacity distribution with higher correlation at the top half, I don't find a negative statistically significant correlation for observations with low levels of fiscal capacity. Hence, the data suggest more a *crowding in* effect as fiscal capacity increases than a crowding out effect as fiscal capacity falls.

In sum, the empirical exercises presented in this section provide support to four theoretical results. I interpret this evidence as support for the relevant role of fiscal capacity in the provision of public liquidity.

6 Final Remarks

In this paper, I study how heterogeneous levels of fiscal capacity impact the ability of a government to provide liquidity. I show that aggregate liquidity shortages happen due to binding financial frictions. In turn, fiscal capacity determines the degree to which government liquidity is subject to these same financial friction. Therefore, when fiscal capacity is sufficiently developed, governments can provide liquidity at will. In contrast, when fiscal capacity is underdeveloped, governments must rely on second-best policies such as foreign reserves accumulation to protect their economies against aggregate liquidity shortages. Additionally, I use an unbalanced panel of 43 countries between 1991 and 2019 to provide empirical support for four different theoretical implications.

I argue that the main mechanism through which the lack of fiscal capacity affects public liquidity provision is through a crowding out channel and, not as previously identified in the literature, by limiting the amount of tax revenue.

The novel result of the crowding out channel has, at least, two policy implications. First, reserves in my theoretical framework are *foreign* in the sense that they are backed up by pledgeable output from other economies, and, unlike the definition of the IMF, there is no need for these assets to be denominated in foreign currency. This result is consistent with Fornaro (2022) who argues that, within a monetary union, fiscal transfers between countries can eliminate sudden stops.Second, tax structure matter for ex-post liquidity provision policies. As argued by Tirole (2002), public liquidity might fail to increase aggregate liquidity if people expect that ensuing tax burden falls into agents that need that liquidity. Hence, ex-post policies require implicit or explicit transfers from liquid to illiquid agents. This is only possible if the tax structure encompasses both type of agents.

Overall, this paper underscores the role that fiscal capacity has on the ability of a government to implement liquidity policies. However, I abstract from how the choice of investments in fiscal capacity interact with the choices of second-best policies. Also, I show that currency mismatch is not a necessary condition for LOLRs to accumulate reserves. An obvious extension to this model would be to include two different goods (tradable and non-tradable) to study how real exchange rate movements and fiscal capacity interact. Plausibly, the existence of different levels of fiscal capacity could even justify the surge of currency mismatches. These relevant and interesting questions that I leave for future research.

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Appendices

A Empirical Appendix

A.1 Data Description

The main constraint behind building the sample of countries was data availability. However, I also excluded countries with less than 5 years of observations, with population less than one million, and I end my analysis in 2019 to abstract from any potential effects coming from the Global COVID Pandemic years. Additionally, I also remove countries that are part of the eurozone any moment during the period of analysis for two reasons. First, data for broad money is not available for each country once they adopt the euro, thus, it would need to be imputed. And, second, the definition of foreign reserves require that these assets are in foreign currency. The day a country transitions from their domestic currency to the euro, all the reserves that were denominated in euros are no longer registered as foreign reserves which creates a potential measurement error in our dependent variable.

Table A.1 presents every variable used, its source, and its explanation. Table A.2 presents each country in the sample, the year of the first observation, and the year corresponding to the last observation, and, lastly, the total number of observations.

A.2 Sets of Control Variables

I build the sets of control variables based on the literature that has aimed at estimating the forces that explain foreign reserves accumulation. In particular, I refer the reader to Aizenman and Lee (2007), Obstfeld et al. (2010), and Ghosh et al. (2017).

I construct four sets of variables that capture different motives to accumulate foreign reserves. The fiscal capacity set is explained in the main document. Below I explain the other three sets: the Traditional set, the Financial Stability Set, and the Mercantalist Set.

The Traditional Set, as suggested by Heller (1966), views reserves as the proper instrument to alleviate adjustment costs to risks emanating from the current account. I include three variables to be part of this model: Imports of goods and services (% of GDP Imports), volatility of Exports (% GDP, 3-year standard deviation), and annual volatility of the exchange rate (standard deviation of monthly growth). Imports captures the dependence of an economy's consumption on international markets while the volatility of exports proxies the safety of the receipts from the world that finance those imports. In turn, the volatility of the exchange rate is used to include in the regression what Heller (1966) called the expenditure-switching adjustment.

The Financial Stability Set groups concerns over shocks to the financial account. As emphasized by Obstfeld et al. (2010), both an internal (from deposits to currency) and external (from domestic to foreign assets) drains can turn into a balance of payment crisis. As in the literature, I use the share of broad money in the economy and it's external short term debt, again relative to the economy, to capture these risks, respectively. Additionally, I include the Chinn-Ito Index (normalized) as a measure of a country's financial openness, dummy variables for a country being an high income as defined by the World Bank, GDP

Table A.1: Variables Dictionary

Variable	Source	Comments
Foreign Reserves (% GDP)	WDI	FI.RES.TOTL.CD
Income Tax Revenue to Individuals (% GDP)	Global Revenue Statistics Database OECD	Total - 1100 Taxes on income, profits and capital gains of individuals
Income Tax Revenue to Businesses (% GDP)	Global Revenue Statistics Database OECD	Total - 1200 Taxes on income, profits and capital gains of corporates
Tax Revenue (% GDP)	Global Revenue Statistics Database OECD	Total - Total Tax Revenue
IIP, Assets, Portfolio investment, Debt securities ($IIP_{A,PF,D}$)	International Financial Statistics (IFS)	[BPM6], US Dollar Million
IIP, Liabilities, Portfolio investment, Debt securities ($IIP_{L,PF,D}$)	International Financial Statistics (IFS)	[BPM6], US Dollar Million
Portfolio Debt Net IIP (% GDP)	Own Calculations	$(IIP_{A,PF,D} - IIP_{A,PF,D}) * 1e6/(GDP) * 100$
Tax Revenue (% GDP)	WDI	GC.TAX.TOTL.GD.ZS
Income Tax Revenue (% TR)	WDI	GC.TAX.YPKG.RV.ZS
Imports (% GDP)	WDI	NE.IMP.GNFS.ZS
Exports Vol. (3-year sd)	WDI	NE.EXP.GNFS.ZS Standard deviation - 3 year moving window
Monthly ER Vol. (Annual sd)	IFS - Nominal Exchange Rates	Standard deviation of monthly percentage variation.
Gross Domestic Product	WDI	NY.GDP.MKTP.CD
Gross Domestic Product Growth	WDI	NY.GDP.MKTP.KD.ZG
Broad Money (% GDP)	WDI, GFD	FM.LBL.BMNY.GD.ZS.
Chinn Ito Index (0-1)	Chinn - Ito index website	Standarized index - Download here
High Income dummy	WDI	Dummy equal to 1 if classified as high income by the World Bank
Hard Peg dummy	Exchange Rate Regime Ilzetzki, Reinhart, and Rogoff Classification	Dummy equal to 1 for values of the Fine index lower or equal to 9 or equal to 11 - Download data here
Soft Peg dummy	Exchange Rate Regime Ilzetzki, Reinhart, and Rogoff Classification	Dummy equal to 1 for values of the Fine index equal to 10 or equal to 12 - Download data here
Short Term Debt (% GDP)	Joint External Debt Hub	The sum of short term 12-Liabilities to BIS banks (cons.), 13-Multilateral loans, 15-Debt securities held by nonresidents, and 18-International debt
Net barter terms of trade index (2000 = 100)	WDI	TT.PRI.MRCH.XD.WD
Domestic Financial Liab. (% GDP)	GFD	Private credit by deposit money banks and other financial institutions to GDP
Private Foreign Liabilities (% GDP)	BIS Reporting Banks	Difference between Total Foreign Liabilities by Nationality and Foreign Liabilities by Nationality - Public Sector - USD Million (stock) over GDP
Public Foreign Liabilities (% GDP)	BIS Reporting Banks (Consolidated), WDI	BIS Reporting Banks - Foreign Liabilities by Nationality - Public Sector - USD Million (stock) over GDP
GDP per capita, PPP (constant 2017 international $)	WDI	NY.GDP.PCAP.PP.KD

Table A.2: Countries in Sample

Country	Isocode-3	Start	End	Observations
Argentina	ARG	1992	2019	28
Australia	AUS	1991	2019	29
Bolivia	BOL	1998	2019	14
Brazil	BRA	2002	2019	18
Cameroon	CMR	2013	2018	6
Canada	CAN	1998	2019	22
Chile	CHL	1998	2019	22
Colombia	COL	1997	2019	23
Costa Rica	CRI	2006	2019	14
Czech Republic	CZE	1994	2019	26
Denmark	DNK	2000	2019	20
Dominican Republic	DOM	2003	2019	17
El Salvador	SLV	2003	2019	17
Ghana	GHA	2008	2019	12
Guatemala	GTM	2006	2019	14
Honduras	HND	2009	2019	11
Hungary	HUN	1996	2019	24
Indonesia	IDN	2003	2019	17
Israel	ISR	1996	2019	24
Jamaica	JAM	2010	2019	10
Japan	JPN	1998	2019	22
Kazakhstan	KAZ	1999	2019	21
Kenya	KEN	2009	2019	11
Korea, Rep.	KOR	1995	2019	25
Malaysia	MYS	2002	2019	18
Mauritius	MUS	2008	2019	12
Mexico	MEX	2003	2019	17
Morocco	MAR	2003	2019	9
New Zealand	NZL	2001	2019	19
Norway	NOR	2000	2019	20
Panama	PAN	1996	2019	24
Paraguay	PRY	2001	2019	17
Peru	PER	1996	2019	24
Philippines	PHL	2002	2019	18
Poland	POL	1998	2019	22
Singapore	SGP	2002	2019	18
South Africa	ZAF	1994	2019	26
Sweden	SWE	2000	2019	20
Switzerland	CHE	1998	2019	21
Trinidad and Tobago	TTO	2012	2019	8
Turkey	TUR	1997	2019	23
United Kingdom	GBR	1998	2019	22
United States	USA	1998	2019	22
Uruguay	URY	2000	2019	20

per capita, and whether the country was implementing during the respective year a hard peg or a soft peg according to Ilzetzki et al. (2019) exchange rate regime index. Hard pegs are countries whose Ilzetzki et al. (2019) Fine index was less or equal to 9 or equal to 11, while soft pegs corresponds to categories 10 or 12.

The Mercantalist Set interprets reserves hoarding as a by-product of a government trying to prevent or slowdown appreciation forces in order to protect the country's export competitiveness. Aizenman and Lee (2007) were one of the first papers to empirically consider the mercantilist motive as a set of explanatory variables behind foreign reserves accumulation. Since there is no straight froward variable that captures government policies motives, the main approach has been to use different measures of a country's exchange rate undervaluation.For instance, Aizenman and Lee (2007) use the *Penn Effect*, Ghosh et al. (2017) consider three different methodologies including non-public IMF assessments of currency undervaluation, and Dominguez (2009) uses the ratio between the Parity Purchasing Power (PPP) conversion factor and the market exchange rate minus one. I follow Cabezas and De Gregorio (2019) who use the terms of trade index and GDP growth as proxies for this motive. The reasoning is that positive shocks to either of these variables is a appreciation force to the real exchange rate. Thus, accumulation of reserves as a response to these shocks reveals a preference of a government to slowdown such appreciation, thus, the mercantilist motive.

A.3 Complementary Material

In this section, I present the summary statistics of the main sample, as well as the complete regressions of the empirical exercise (Tables A.3-A.7).

Table A.3: Summary statistics

	Mean	SD	Min	Max	N
Dependent Variables					
Foreign Reserves exc. Gold (% GDP, log)	-2.1	0.9	-5.5	0.1	827
Fiscal Capacity Model					
Income Tax People (% TR, log)	-2.0	1.0	-6.7	-0.6	827
Income Tax Business (% TR, log)	-2.0	0.5	-3.4	-0.5	827
Tax Revenue (% GDP, log)	3.1	0.4	2.3	3.9	827
Private Foreign Liabilities (% GDP, log)	-2.0	1.1	-8.0	1.0	827
Public Foreign Liabilities (% GDP, log)	-3.9	1.2	-14.3	-1.5	827
Portfolio Debt Net IIP (% GDP)	-5.5	30.9	-90.1	166.3	827
Traditional Model					
Imports (% GDP, log)	3.5	0.5	1.8	5.3	819
Exports Vol. (log, 3-year sd)	0.1	0.1	0.0	0.5	818
Monthly ER Vol. (Annual sd)	0.0	0.0	0.0	0.2	827
Financial Sta. Model					
Broad Money (% GDP, log)	4.1	0.5	2.1	5.5	803
Chinn Ito Index (0-1)	0.7	0.3	0.0	1.0	824
Hard Peg dummy	0.6	0.5	0.0	1.0	827
Soft Peg dummy	0.2	0.4	0.0	1.0	827
Short Term Debt (% GDP, log)	-2.2	1.0	-5.0	0.3	827
GDP per capita, PPP (log)	9.9	0.8	8.1	11.5	827
Mercantalist Model					
Terms of Trade Index	114.5	31.3	50.2	258.2	781
GDP Growth	3.4	3.0	-10.9	19.0	825

A.4 Robustness Exercises

In this section, I present robustness exercises of the empirical section. The exercises are the same with the exception that I proxy fiscal capacity with total income tax revenue as percentage of total taxation. The source is the Word Development Indicators dataset. This variable change allows me to increase the sample to 73 countries. Table A.8 presents the full sample, Table A.9 the summary statistics specific to this WDI sample, and Tables A.10-A.12 the robustness exercises.

Table A.4: Foreign Reserves and Fiscal Capacity - Full Regression

	1991-2019		Pre-GFC		Post-GFC	
	(1)	(2)	(3)	(4)	(5)	(6)
Income Tax People (% TR, log)	-0.34***	-0.18**	-0.36***	-0.18*	-0.31**	-0.05
	(0.11)	(0.08)	(0.13)	(0.09)	(0.13)	(0.07)
Income Tax Business (% TR, log)	0.46*	0.47***	0.61**	0.58**	0.39	0.41**
	(0.23)	(0.14)	(0.23)	(0.22)	(0.26)	(0.16)
Tax Revenue (% GDP, log)	0.72*	0.85**	0.57	0.81*	0.84*	0.87**
	(0.40)	(0.33)	(0.42)	(0.47)	(0.48)	(0.35)
Private Foreign Liabilities (% GDP, log)	0.06	-0.08	-0.02	0.13	0.14	-0.32**
	(0.12)	(0.11)	(0.13)	(0.18)	(0.15)	(0.12)
Public Foreign Liabilities (% GDP, log)	0.17**	0.12*	0.20**	0.16	0.19**	0.08
	(0.07)	(0.07)	(0.08)	(0.11)	(0.09)	(0.07)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Imports (% GDP, log)		0.55***		0.33		0.78***
		(0.18)		(0.26)		(0.19)
Exports Vol. (log, 3-year sd)		0.38		0.48		0.44
		(0.48)		(0.66)		(0.77)
Monthly ER Vol. (Annual sd)		5.36		2.96		7.74**
		(3.30)		(3.77)		(3.65)
Broad Money (% GDP, log)		0.31*		0.19		0.44**
		(0.16)		(0.23)		(0.17)
Chinn Ito Index (0-1)		0.15		-0.10		0.32
		(0.23)		(0.32)		(0.24)
Hard Peg dummy		0.62***		0.61*		0.77***
		(0.21)		(0.35)		(0.21)
Soft Peg dummy		0.61***		0.49		0.84***
		(0.21)		(0.32)		(0.19)
Short Term Debt (% GDP, log)		-0.02		-0.31		0.23*
		(0.13)		(0.21)		(0.14)
GDP per capita, PPP (log)		-0.60**		-0.48		-0.83***
		(0.26)		(0.39)		(0.24)
Terms of Trade Index		-0.00***		-0.01*		-0.00*
		(0.00)		(0.00)		(0.00)
GDP Growth		0.01		-0.00		0.01
		(0.01)		(0.02)		(0.01)
Observations	826	747	289	258	425	393
R^2	0.42	0.66	0.39	0.61	0.41	0.76
Countries	44	43	37	37	44	41

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is total foreign reserves, excluding gold (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression.

Table A.5: Foreign Reserves and Fiscal Capacity by Percentiles - Full Regression

	Fiscal Capacity Set						+ Other Sets					
	(1) Full Sample	(2) ≥p10	(3) ≥p20	(4) ≥p30	(5) ≥p40	(6) ≥p50	(7) Full Sample	(8) ≥p10	(9) ≥p20	(10) ≥p30	(11) ≥p40	(12) ≥p50
Income Tax People (% TR, log)	-0.34***	-0.56***	-0.80***	-1.07***	-1.25***	-1.32**	-0.18**	-0.32**	-0.33**	-0.48*	-0.46	-0.56
	(0.11)	(0.19)	(0.24)	(0.29)	(0.40)	(0.50)	(0.08)	(0.13)	(0.16)	(0.25)	(0.35)	(0.41)
Income Tax Business (% TR, log)	0.46*	0.42*	0.44*	0.41*	0.40	0.39	0.47***	0.47***	0.46***	0.43***	0.46***	0.41**
	(0.23)	(0.23)	(0.23)	(0.23)	(0.27)	(0.37)	(0.14)	(0.14)	(0.15)	(0.14)	(0.13)	(0.16)
Tax Revenue (% GDP, log)	0.72*	0.78*	0.97*	1.18*	1.17*	1.33	0.85**	0.90**	0.80**	0.79**	0.87***	1.03***
	(0.40)	(0.44)	(0.51)	(0.59)	(0.66)	(0.85)	(0.33)	(0.34)	(0.31)	(0.31)	(0.32)	(0.37)
Private Foreign Liabilities (% GDP, log)	0.06	0.05	0.03	0.03	0.04	0.05	-0.08	-0.13	-0.16	-0.20*	-0.19	-0.11
	(0.12)	(0.13)	(0.14)	(0.16)	(0.17)	(0.20)	(0.11)	(0.11)	(0.11)	(0.12)	(0.12)	(0.15)
Public Foreign Liabilities (% GDP, log)	0.17**	0.20***	0.18**	0.14	0.11	0.10	0.12*	0.13*	0.12	0.05	0.01	-0.02
	(0.07)	(0.07)	(0.08)	(0.10)	(0.12)	(0.13)	(0.07)	(0.07)	(0.08)	(0.10)	(0.10)	(0.12)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Imports (% GDP, log)							0.55***	0.64***	0.70***	0.70***	0.81***	0.85***
							(0.18)	(0.18)	(0.18)	(0.19)	(0.21)	(0.25)
Exports Vol. (log, 3-year sd)							0.38	0.08	0.06	-0.25	0.99	1.23
							(0.48)	(0.55)	(0.79)	(0.96)	(0.90)	(0.88)
Monthly ER Vol. (Annual sd)							5.36	5.12	8.79**	10.78**	10.90**	10.58**
							(3.30)	(3.23)	(4.04)	(4.59)	(4.71)	(4.85)
Broad Money (% GDP, log)							0.31*	0.31*	0.35*	0.31	0.18	0.22
							(0.16)	(0.17)	(0.18)	(0.20)	(0.20)	(0.24)
Chinn Ito Index (0-1)							0.15	0.12	0.28	0.14	-0.08	-0.19
							(0.23)	(0.29)	(0.29)	(0.27)	(0.30)	(0.32)
Hard Peg dummy							0.62***	0.50**	0.53**	0.56**	0.49**	0.46**
							(0.21)	(0.22)	(0.24)	(0.22)	(0.21)	(0.20)
Soft Peg dummy							0.61***	0.57**	0.63***	0.62***	0.54**	0.56**
							(0.21)	(0.21)	(0.22)	(0.22)	(0.21)	(0.21)
Short Term Debt (% GDP, log)							-0.02	0.04	0.03	0.07	0.03	-0.04
							(0.13)	(0.13)	(0.13)	(0.15)	(0.14)	(0.13)
GDP per capita, PPP (log)							-0.60**	-0.60**	-0.62**	-0.59**	-0.50*	-0.50*
							(0.26)	(0.26)	(0.26)	(0.25)	(0.27)	(0.26)
Terms of Trade Index							-0.00***	-0.01***	-0.00**	-0.00	-0.01**	-0.00
							(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
GDP Growth							0.01	0.01	0.01	-0.00	-0.01	-0.01
							(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Observations	826	749	666	584	501	418	747	673	590	513	441	366
R^2	0.42	0.45	0.51	0.59	0.60	0.57	0.66	0.68	0.72	0.77	0.78	0.78
Countries	44	41	39	36	31	28	43	40	38	35	29	27

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is total foreign reserves, excluding gold (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression. ≥pYY refers to estimates of a regression that includes observations whose fiscal capacity is equal or above YY% percentile.

Table A.6: Multiple Equilibria and Fiscal Capacity - Full Regression

	(1)	(2)
Portfolio Debt Net IIP (% GDP)	-0.001	-0.002
	(0.003)	(0.002)
inter. with Quartile 2 FC Dummy	0.012***	0.013***
	(0.004)	(0.003)
inter. with Quartile 3 FC Dummy	0.008*	0.011***
	(0.004)	(0.003)
inter. with Quartile 4 FC Dummy	0.015***	0.012***
	(0.004)	(0.002)
Income Tax People (% TR, log)	-0.219***	-0.104
	(0.077)	(0.071)
Quartile 2 FC (dummy)	0.384***	0.172
	(0.111)	(0.148)
Quartile 3 FC (dummy)	0.357*	0.162
	(0.188)	(0.201)
Imports (% GDP, log)		0.564***
		(0.155)
Exports Vol. (log, 3-year sd)		-0.136
		(0.497)
Monthly ER Vol. (Annual sd)		4.932*
		(2.862)
Broad Money (% GDP, log)		0.299*
		(0.149)
Chinn Ito Index (0-1)		0.162
		(0.211)
Hard Peg dummy		0.457**
		(0.206)
Soft Peg dummy		0.513**
		(0.210)
Short Term Debt (% GDP, log)		0.139
		(0.107)
GDP per capita, PPP (log)		-0.499**
		(0.217)
Terms of Trade Index		-0.002*
		(0.001)
GDP Growth		0.003
		(0.011)
Income Tax Business (% TR, log)	0.431**	0.396***
	(0.213)	(0.139)
Tax Revenue (% GDP, log)	0.908**	0.978***
	(0.417)	(0.309)
Private Foreign Liabilities (% GDP, log)	0.117	-0.135
	(0.121)	(0.090)
Quartile 4 FC (dummy)	-0.483**	-0.520**
	(0.209)	(0.195)
Public Foreign Liabilities (% GDP, log)	0.069	0.037
	(0.063)	(0.059)
Observations	826	747
R^2	0.57	0.71
Countries	44	43

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is Total foreign reserves, excluding Gold (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression.

Table A.7: Crowding Out and Fiscal Capacity - Full Regression

	(1)	(2)	(3)	(4)
Public Foreign Liabilities (% GDP, log)	0.41***	0.13***	0.14*	-0.03
	(0.08)	(0.04)	(0.08)	(0.04)
inter. with Quartile 2 FC Dummy			0.31	0.03
			(0.19)	(0.09)
inter. with Quartile 3 FC Dummy			0.37***	0.22***
			(0.13)	(0.06)
inter. with Quartile 4 FC Dummy			0.53***	0.36**
			(0.18)	(0.14)
Income Tax People (% TR, log)	-0.06	-0.05	0.00	0.08*
	(0.10)	(0.04)	(0.09)	(0.04)
Quartile 2 FC (dummy)			1.46	0.17
			(0.88)	(0.36)
Quartile 3 FC (dummy)			1.27**	0.66**
			(0.49)	(0.27)
Quartile 4 FC (dummy)			2.17***	1.04**
			(0.60)	(0.50)
Reserves + Portfolio Debt Net IIP (% GDP)	0.00*	-0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Imports (% GDP, log)		0.47***		0.46***
		(0.09)		(0.10)
Exports Vol. (log, 3-year sd)		-0.76		-0.86*
		(0.47)		(0.47)
Monthly ER Vol. (Annual sd)		-2.47		-1.77
		(1.47)		(1.34)
Broad Money (% GDP, log)		0.00		0.01
		(0.11)		(0.11)
Chinn Ito Index (0-1)		-0.01		0.12
		(0.13)		(0.12)
Hard Peg dummy		0.04		-0.07
		(0.11)		(0.13)
Soft Peg dummy		0.05		0.03
		(0.10)		(0.12)
Short Term Debt (% GDP, log)		0.73***		0.73***
		(0.06)		(0.06)
GDP per capita, PPP (log)		-0.27**		-0.23*
		(0.13)		(0.11)
Observations	825	789	825	789
R^2	0.41	0.83	0.47	0.85
Countries	44	43	44	43

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is Private Foreign Liabilities (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression.

Table A.8: Countries in Sample with WDI data

Country	Isocode-3	Start	End	Observations	Obs in Main Sample
Angola	AGO	2011	2019	9	N/A
Albania	ALB	2012	2019	8	N/A
Argentina	ARG	1992	2019	28	28
Armenia	ARM	2006	2019	14	N/A
Australia	AUS	1991	2019	29	29
Bulgaria	BGR	2006	2019	14	N/A
Bosnia and Herzegovina	BIH	2006	2019	14	N/A
Belarus	BLR	1997	2019	23	N/A
Bolivia	BOL	1998	2008	8	14
Brazil	BRA	2011	2019	9	18
Botswana	BWA	2007	2019	13	N/A
Canada	CAN	1998	2019	22	22
Switzerland	CHE	1998	2019	21	21
Chile	CHL	1998	2019	22	22
China	CHN	2006	2019	14	N/A
Cameroon	CMR	2013	2018	6	6
Colombia	COL	1999	2019	15	23
Costa Rica	CRI	2006	2019	14	14
Czech Republic	CZE	1994	2019	26	26
Denmark	DNK	2000	2019	20	20
Dominican Republic	DOM	2003	2019	17	17
Egypt, Arab Rep.	EGY	2005	2016	12	N/A
United Kingdom	GBR	1998	2019	22	22
Georgia	GEO	2001	2019	19	N/A
Ghana	GHA	2008	2019	12	12
Guatemala	GTM	2006	2019	14	14
Honduras	HND	2009	2016	8	11
Croatia	HRV	2000	2019	20	N/A
Hungary	HUN	1996	2019	24	24
Indonesia	IDN	2002	2019	15	17
India	IND	1997	2019	23	N/A
Israel	ISR	1996	2019	24	24
Jamaica	JAM	2010	2019	10	10
Jordan	JOR	1997	2016	13	N/A
Kazakhstan	KAZ	1998	2019	17	21
Cambodia	KHM	2011	2019	9	N/A
Korea, Rep.	KOR	1995	2019	25	25
Sri Lanka	LKA	2012	2019	7	N/A
Morocco	MAR	2003	2019	9	9
Moldova	MDA	2002	2019	13	N/A
Madagascar	MDG	2006	2019	12	N/A
Mexico	MEX	2009	2019	11	17
North Macedonia	MKD	2011	2019	5	N/A
Myanmar	MMR	2015	2019	5	N/A
Mongolia	MNG	2011	2019	9	N/A
Mozambique	MOZ	2011	2019	9	N/A
Mauritius	MUS	2008	2019	12	12
Malaysia	MYS	2002	2019	18	18
Namibia	NAM	2001	2019	16	N/A
Nicaragua	NIC	2009	2019	9	N/A
Norway	NOR	2000	2019	20	20
New Zealand	NZL	2002	2019	18	19
Panama	PAN	2015	2019	5	24
Peru	PER	1996	2019	24	24
Philippines	PHL	2002	2019	18	18
Poland	POL	1998	2019	22	22
Paraguay	PRY	2006	2019	14	17
Romania	ROU	1996	2019	24	N/A
Russian Federation	RUS	2000	2019	19	N/A
Saudi Arabia	SAU	2013	2019	7	N/A
Singapore	SGP	2002	2019	18	18
El Salvador	SLV	2000	2019	20	17
Serbia	SRB	2009	2019	11	N/A
Sweden	SWE	2000	2019	20	20
Thailand	THA	2001	2019	19	N/A
Trinidad and Tobago	TTO	2012	2019	8	8
Turkey	TUR	1997	2019	14	23
Tanzania	TZA	2010	2018	9	N/A
Ukraine	UKR	2001	2019	19	N/A
Uruguay	URY	1992	2019	28	20
United States	USA	1998	2019	22	22
Uzbekistan	UZB	2015	2019	5	N/A
South Africa	ZAF	1991	2019	29	26

Table A.9: Summary statistics - WDI

	Mean	SD	Min	Max	N
Dependent Variables					
Foreign Reserves exc. Gold (% GDP, log)	-2.0	0.9	-5.5	0.1	1,141
Fiscal Capacity Model					
Income Tax WDI (% TR, log)	3.1	0.7	0.0	4.3	1,141
Tax Revenue (% GDP, log)	2.8	0.4	0.9	3.6	1,141
Private Foreign Liabilities (% GDP, log)	-2.2	1.0	-8.0	0.6	1,141
Public Foreign Liabilities (% GDP, log)	-4.0	1.3	-16.0	-1.5	1,141
Portfolio Debt Net IIP (% GDP)	-5.2	26.4	-90.1	166.3	1,141
Traditional Model					
Imports (% GDP, log)	3.6	0.5	1.8	5.3	1,130
Exports Vol. (log, 3-year sd)	0.1	0.1	0.0	0.6	1,128
Monthly ER Vol. (Annual sd)	2.9	95.9	0.0	3,238.3	1,141
Financial Sta. Model					
Broad Money (% GDP, log)	4.0	0.5	2.1	5.3	1,119
Chinn Ito Index (0-1)	0.6	0.4	0.0	1.0	1,127
Hard Peg dummy	0.7	0.5	0.0	1.0	1,136
Soft Peg dummy	0.2	0.4	0.0	1.0	1,136
Short Term Debt (% GDP, log)	-2.5	1.0	-6.3	0.2	1,141
GDP per capita, PPP (log)	9.7	0.8	7.0	11.5	1,141
Mercantalist Model					
Terms of Trade Index	114.5	32.7	50.2	283.2	1,086
GDP Growth	3.7	3.4	-15.1	19.0	1,139

Table A.10: Foreign Reserves and Fiscal Capacity - WDI Data

	1991-2019		Pre-GFC		Post-GFC	
	(1)	(2)	(3)	(4)	(5)	(6)
Income Tax WDI (% TR, log)	-0.32**	-0.22**	-0.31*	-0.16	-0.30**	-0.19*
	(0.14)	(0.09)	(0.16)	(0.10)	(0.15)	(0.11)
Tax Revenue (% GDP, log)	0.41	0.06	0.49	0.14	0.37	-0.08
	(0.33)	(0.26)	(0.53)	(0.39)	(0.30)	(0.25)
Private Foreign Liabilities (% GDP, log)	0.09	0.20*	0.03	0.34	0.13	0.10
	(0.11)	(0.10)	(0.15)	(0.23)	(0.11)	(0.13)
Public Foreign Liabilities (% GDP, log)	0.09	0.08	0.16	0.05	0.06	0.07
	(0.07)	(0.06)	(0.11)	(0.14)	(0.06)	(0.06)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01**	0.01	0.01***	0.01***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)
Imports (% GDP, log)		0.44***		0.40*		0.50***
		(0.16)		(0.22)		(0.15)
Exports Vol. (log, 3-year sd)		1.14*		1.28*		0.42
		(0.61)		(0.71)		(0.90)
Monthly ER Vol. (Annual sd)		-0.00*		-0.00		5.14**
		(0.00)		(0.00)		(2.44)
Broad Money (% GDP, log)		0.46***		0.49***		0.47***
		(0.12)		(0.13)		(0.13)
Chinn Ito Index (0-1)		0.06		-0.15		0.21
		(0.21)		(0.27)		(0.21)
Hard Peg dummy		0.81***		1.00***		0.79**
		(0.24)		(0.35)		(0.31)
Soft Peg dummy		0.84***		1.02***		0.80**
		(0.29)		(0.34)		(0.31)
Short Term Debt (% GDP, log)		-0.21		-0.30		-0.10
		(0.14)		(0.26)		(0.12)
GDP per capita, PPP (log)		-0.26*		-0.21		-0.31**
		(0.14)		(0.26)		(0.13)
Terms of Trade Index		0.00		0.00		0.00
		(0.00)		(0.00)		(0.00)
GDP Growth		0.01		0.02		0.00
		(0.01)		(0.01)		(0.01)
Observations	1141	1036	340	302	655	608
R^2	0.38	0.59	0.32	0.59	0.35	0.58
Countries	73	70	51	51	72	68

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is Total Foreign Reserves, excluding gold (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression.

Table A.11: Foreign Reserves and Fiscal Capacity by Percentiles - WDI

	Fiscal Capacity Set						+ Other Sets					
	(1) Full Sample	(2) ≥p10	(3) ≥p20	(4) ≥p30	(5) ≥p40	(6) ≥p50	(7) Full Sample	(8) ≥p10	(9) ≥p20	(10) ≥p30	(11) ≥p40	(12) ≥p50
Income Tax WDI (% TR, log)	-0.32**	-0.46**	-0.59**	-0.67**	-0.73*	-0.91*	-0.22**	-0.24*	-0.22	-0.31	-0.41*	-0.66**
	(0.14)	(0.20)	(0.24)	(0.30)	(0.38)	(0.51)	(0.09)	(0.13)	(0.15)	(0.20)	(0.24)	(0.30)
Tax Revenue (% GDP, log)	0.41	0.62*	0.65*	0.67	0.66	0.62	0.06	0.26	0.21	0.21	0.19	0.21
	(0.33)	(0.36)	(0.39)	(0.41)	(0.43)	(0.46)	(0.26)	(0.29)	(0.29)	(0.31)	(0.31)	(0.32)
Private Foreign Liabilities (% GDP, log)	0.09	0.11	0.10	0.11	0.07	0.07	0.20*	0.19*	0.21**	0.23*	0.17	0.09
	(0.11)	(0.11)	(0.12)	(0.14)	(0.16)	(0.19)	(0.10)	(0.10)	(0.10)	(0.12)	(0.13)	(0.14)
Public Foreign Liabilities (% GDP, log)	0.09	0.02	0.02	0.01	0.02	0.02	0.08	0.02	0.02	0.02	0.02	0.01
	(0.07)	(0.07)	(0.07)	(0.07)	(0.09)	(0.11)	(0.06)	(0.05)	(0.05)	(0.06)	(0.07)	(0.09)
Portfolio Debt Net IIP (% GDP)	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01***	0.01**	0.01**	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Imports (% GDP, log)							0.44***	0.54***	0.58***	0.59***	0.63***	0.78***
							(0.16)	(0.15)	(0.17)	(0.18)	(0.19)	(0.23)
Exports Vol. (log, 3-year sd)							1.14*	1.17*	1.30*	1.15	1.33	2.03
							(0.61)	(0.60)	(0.74)	(0.88)	(0.98)	(1.25)
Monthly ER Vol. (Annual sd)							-0.00*	-0.00***	2.60	3.93	6.14**	4.73
							(0.00)	(0.00)	(2.20)	(2.74)	(2.85)	(4.11)
Broad Money (% GDP, log)							0.46***	0.38***	0.41***	0.43***	0.47***	0.34**
							(0.12)	(0.10)	(0.11)	(0.11)	(0.11)	(0.14)
Chinn Ito Index (0-1)							0.06	-0.05	-0.05	-0.06	-0.04	-0.17
							(0.21)	(0.20)	(0.22)	(0.25)	(0.26)	(0.29)
Hard Peg dummy							0.81***	0.74***	0.82***	0.82***	0.86***	0.76***
							(0.24)	(0.21)	(0.24)	(0.23)	(0.24)	(0.24)
Soft Peg dummy							0.84***	0.79***	0.84***	0.83***	0.87***	0.76***
							(0.29)	(0.25)	(0.26)	(0.25)	(0.25)	(0.25)
Short Term Debt (% GDP, log)							-0.21	-0.16	-0.19	-0.21	-0.17	-0.06
							(0.14)	(0.13)	(0.14)	(0.17)	(0.21)	(0.24)
GDP per capita, PPP (log)							-0.26*	-0.30**	-0.30*	-0.27	-0.16	-0.14
							(0.14)	(0.14)	(0.16)	(0.17)	(0.17)	(0.20)
Terms of Trade Index							0.00	-0.00	-0.00	-0.00	-0.00	-0.00
							(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
GDP Growth							0.01	0.01	0.02*	0.02	0.02*	0.03*
							(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Observations	1141	1038	924	810	696	581	1036	934	836	739	635	526
R^2	0.38	0.40	0.41	0.43	0.47	0.51	0.59	0.62	0.63	0.64	0.67	0.71
Countries	73	71	66	61	55	49	70	68	63	57	52	45

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is total foreign reserves, excluding gold (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression. ≥pYY refers to estimates of a regression that includes observations whose fiscal capacity is equal or above YY% percentile.

Table A.12: Crowding Out and Fiscal Capacity - WDI Data

	(1)	(2)	(3)	(4)
Public Foreign Liabilities (% GDP, log)	0.34***	0.38***	0.11***	0.16***
	(0.05)	(0.06)	(0.04)	(0.05)
inter. with Quartile 2 FC Dummy		-0.15*		-0.17***
		(0.09)		(0.06)
inter. with Quartile 3 FC Dummy		-0.01		-0.09
		(0.11)		(0.06)
inter. with Quartile 4 FC Dummy		0.04		0.08
		(0.10)		(0.07)
Income Tax WDI (% TR, log)	-0.02	-0.04	-0.08	-0.14*
	(0.08)	(0.19)	(0.05)	(0.08)
Reserves + Portfolio Debt Net IIP (% GDP)	0.00*	0.00**	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Quartile 2 FC (dummy)		-0.53		-0.55**
		(0.32)		(0.27)
Quartile 3 FC (dummy)		-0.17		-0.30
		(0.48)		(0.26)
Quartile 4 FC (dummy)		0.26		0.45
		(0.42)		(0.31)
Imports (% GDP, log)			0.26***	0.29***
			(0.06)	(0.06)
Exports Vol. (log, 3-year sd)			-0.37	-0.42
			(0.36)	(0.36)
Monthly ER Vol. (Annual sd)			0.00**	0.00***
			(0.00)	(0.00)
Broad Money (% GDP, log)			0.02	0.01
			(0.08)	(0.08)
Chinn Ito Index (0-1)			0.07	0.09
			(0.10)	(0.10)
Hard Peg dummy			-0.00	-0.00
			(0.07)	(0.06)
Soft Peg dummy			0.04	0.07
			(0.07)	(0.08)
Short Term Debt (% GDP, log)			0.70***	0.69***
			(0.05)	(0.05)
GDP per capita, PPP (log)			-0.18*	-0.15
			(0.09)	(0.09)
Observations	1136	1136	1082	1082
R^2	0.49	0.51	0.81	0.82
Countries	73	73	70	70

Note: *** p<0.01, ** p<0.05, * p<0.10. The dependent variable is Private Foreign Liabilities (% GDP, log units). Standard errors in parenthesis, clustered by country. Time and income group fixed effects are not reported but are included in every regression.

B Technical Appendix

B.1 Laissez Faire Equilibria

I find the set of Laissez Faire Equilibria through backward induction. I start at period $t = 1$ since contracts are designed for banking entrepreneurs to abide by requiring satisfying (14).

A given entrepreneur's period 1 optimization problem can be rewritten as follows:

$$\begin{aligned}
&\underset{\{c_1^s, M_1^s, \phi_1^s\}}{\text{Maximize}} \ \big[1-\gamma_1^s\big]c_1^s + \big[\rho_1 - \gamma_1^s\big]j^s + \gamma_1^s x_1^s \\
&\text{subject to: } j^s = min\{\frac{M_1^s}{1-\phi_1^s}, i\} \\
&\big[\gamma_1^s \phi_1^s - \rho_0\big]j^s \leq 0 \\
&c_1^s + M_1^s \leq x_1^s \\
&\{c_1^s, M_1^s, \phi_1^s\} \ non-negative, \text{ given } \{x_1^s, i\}
\end{aligned}$$

I start by solving this problem for a boom For a given $\{x_1^L, i\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 1\}$ is a optimal answer with payoff $\big[\rho_1 - \gamma_1^L\big]i + x_1^L$. To see this, first, now that this solution is feasible since $\gamma_1^L i < \rho_0 i$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^s}, \hat{M_1^s}, \hat{\phi_1^s}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 > \gamma_1^L$ in this scenario, $\hat{j^L} \leq i$ and $\hat{C_1^L} \leq x_1^L$ due to feasibility, and $\gamma_1^L < \rho_1$ as an assumption. Below the optimal strategies for an entrepreneur during a boom.

$$j^L = \Big\{ i \qquad \text{for all } x_1^L \geq 0$$

$$c_1^L = \Big\{ x_1^L \qquad \text{for all } x_1^L \geq 0$$

$$M_1^L = \Big\{ 0 \qquad \text{for all } x_1^L \geq 0$$

$$l_1^L = \Big\{ \gamma_1^L i \qquad \text{for all } x_1^L \geq 0$$

$$\rho_0 j^L - l_1^L = \Big\{ \big[\rho_0 - \gamma_1^L\big]i \qquad \text{for all } x_1^L \geq 0$$

$$C_{1,2}^L = \Big\{ \big[\rho_1 - \gamma_1^L\big]i + x_1^L \qquad \text{for all } x_1^L \geq 0$$

I continue by solving this problem for a stress period For a given $\{x_1^H, i\}$, recall that $\gamma_1^H > \rho_0$ which implies that $\phi_1^H = 1$ is no longer possible (no *finance as you go*). Moreover, $\gamma_1^H > 1$ which implies that it is better to lend at international markets than to consume, thus, c_1^H is equal to zero. **Consider first the scenario where** $x_1^H \geq i\big[1 - \frac{\rho_0}{\gamma_1^H}\big]$**.** I show that, in this scenario, $\{c_1^H = 0, M_1^H = i\big[1 - \frac{\rho_0}{\gamma_1^H}\big], \phi_1^H = \frac{\rho_0}{\gamma_1^H}\}$ is an optimal answer with payoff $\big[\rho_1 - \gamma_1^H\big]i + \gamma_1^H x_1^H$. To see this, first, i show feasibility where $j^H = \phi_1^H j^H + M_1^H =$

$\frac{\rho_0}{\gamma_1^L}i + i\big[1-\frac{\rho_0}{\gamma_1^H}\big] = i$ which is less or equal to i. Additionally, $\big[\gamma_1^H\phi_1^H - \rho_0\big]j^s = \big[\gamma_1^H\frac{\rho_0}{\gamma_1^H} - \rho_0\big]i$ which is equal to zero. Lastly, $c_1^H + M_1^H = 0 + i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \leq x_1^H$ by assumption for this scenario. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^H}, \hat{M_1^H}, \hat{\phi_1^H}, \hat{j^H}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 < \gamma_1^H$ in this scenario, $\hat{j^L} \leq i$ due to feasibility. **Consider now subgames when** $x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big]$. I show that $\{c_1^H = 0, M_1^H = x_1^H, \phi_1^H = \frac{\rho_0}{\gamma_1^H},\ j^H = \frac{x_1^H}{1-\frac{\rho_0}{\gamma_1^H}}\}$ is an optimal answer with payoff $\frac{\rho_1-\rho_0}{\gamma_1^H-\rho_0}\gamma_1^H x_1^H$. To see this, first, I show feasibility where $j^H = \phi_1^H j^H + M_1^H = \frac{\rho_0}{\gamma_1^L}j^H + x_1^H \rightarrow j^H = \frac{x_1^H}{1-\frac{\rho_0}{\gamma_1^H}}$ which is less or equal to i because $x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big]$. Additionally, $\big[\gamma_1^H\phi_1^H - \rho_0\big]j^H = \big[\gamma_1^H\frac{\rho_0}{\gamma_1^H} - \rho_0\big]j^H$ which is equal to zero. Lastly, $c_1^H + M_1^H = 0 + x_1^H \leq x_1^H$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^H}, \hat{M_1^H}, \hat{\phi_1^H},\ \hat{j^H}\}$ that is feasible and produces a strictly greater payoff. First, note that since this candidate is feasible then $\hat{j^H} = \hat{\phi_1^H}\hat{j^H} + \hat{M_1^H} \leq \frac{\rho_0}{\gamma_1^L}j^H + x_1^H \rightarrow \hat{j^H} \leq \frac{x_1^H}{1-\frac{\rho_0}{\gamma_1^H}}$. Thus, any feasible solution cannot generate a reinvestment scale greater than our optimal choice. This finding together with $1 < \gamma_1^H$ and $\hat{c_1^H}$ non-negative imply that no feasible solution produces a greater payoff.

Below the optimal strategies for an entrepreneur during a stress period.

$$j^H = \begin{cases} \frac{x_1^H}{1-\frac{\rho_0}{\gamma_1^H}} & \text{when } x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \\ i & \text{when } x_1^H \geq i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$c_1^H = \begin{cases} 0 & \text{for all } x_1^L \geq 0 \end{cases}$$

$$M_1^H = \begin{cases} x_1^H & \text{when } x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \\ i\big[1-\frac{\rho_0}{\gamma_1^H}\big] & \text{when } x_1^H \geq i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$l_1^H = \begin{cases} \rho_0\frac{x_1^H}{1-\frac{\rho_0}{\gamma_1^H}} & \text{when } x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \\ \rho_0 i & \text{when } x_1^H \geq i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

$$C_{1,2}^H = \begin{cases} \frac{\rho_1-\rho_0}{\gamma_1^H-\rho_0}\gamma_1^H x_1^H & \text{when } x_1^H < i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \\ \big[\rho_1 - \gamma_1^H\big]i + \gamma_1^H x_1^H & \text{when } x_1^H \geq i\big[1-\frac{\rho_0}{\gamma_1^H}\big] \end{cases}$$

Recall that a given entrepreneur's period 0 optimization problem is as follows:

$$\begin{aligned}
\underset{\{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}}{\text{Maximize}} \quad & c_0 + \alpha\big[C_{1,2}^L - l_0^L\big] + (1-\alpha)\big[C_{1,2}^H\big] \\
\text{subject to: } & \alpha\big[l_0^L + d_f^L i\big](1-\alpha)\big[d_f^H i\big] = i - M_0 \\
& d_f^s i + d_e^s i = \pi i \\
& l_0^L \,\epsilon \big[0, \rho_0 j^L - l_1^L \gamma_1^L\big] \\
& A - c_0 - M_0 = x_A \\
& d_e^s i + x_A = x_1^s \\
& \{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\} \text{ non-negative, given the optimal} \\
& \text{functions at date-1 that depend on } \{x_1^L, x_1^H\}
\end{aligned}$$

Could c_0 be positive? No, it can not. Since entrepreneurs are risk neutral, they are always better off to postpone consumption until $t = 1$ after they observe the aggregate shock. If it is a boom, they can consume, while if there is market *stress*, they can lend with a higher return. Either option is at least as good as consuming that unit at $t = 0$ **Could x_A be positive?** A positive x_A increases insurance x_1^s for both states while it sacrifices investment. However, entrepreneurs have another way to accumulate liquidity. That is, by allocating to the project a share of safe cash flow. Since $\pi > 1 - \frac{\rho_0}{\gamma_H^L}$, this cash flow together with the maximum amount of funding liquidity is enough to reach full-scale reinvestment even in stress periods. Additionally, x_A is not state-contingent so it is even more expensive in terms of investment scale. Thus, even in scenarios where x_1^s is positive, there is no need for x_A to be positive. Additionally, following Assumption 1, projects generate have a positive net present value. This confirms that entrepreneurs invest all their net worth in the project. Therefore, M_0 is equal to A. **Should l_0^L be the maximum possible?** Consider the case where reinvestment is only possible in booms since these are long-term claims contigent on a realized boom. If you take the derivative of the objective function relative to l_0^L, the sign will depend on the term of $\alpha(\rho_1 - \gamma_1^L) - (1-\pi)$. By Assumption (1), this term is positive. Then, contracts at period-0 load up in long-term claims for booms. This result is due to the fact that even if no reinvestment is done during stress periods, projects generate a sufficiently high return that it is worth to load up in long-term contingent debt. I have shown that $\{c_0 = 0,\ x_A = 0,\ M_0 = A,\ l_0^L = \rho_0 j^L - l_1^L\}$. What is left to determine is set $\{d_f^L i,\ d_f^H i\}$ which in turn defines the initial investment scale. Note that with x_A equal to zero, then $x_1^s = d_e^s i$. I define $\bar{x_1}^s$ equal to $\frac{x_1^s}{i} = \bar{x_1}^s$ which is the amount of liquidity hoarding by unit of initial investment. Rewriting foreign lenders participation constraint I get that the initial scale is given by

$$i(\bar{x_1}^L, \bar{x_1}^H) = \frac{A}{1 - \pi - \alpha(\rho_0 - \gamma_1^L) + \alpha\bar{x_1}^L + (1-\alpha)\bar{x_1}^H}$$

Recall that $\bar{x_1}^S$ lies between zero and π. Note that the initial investment falls when insurance for in any state increases. The objective function of an entrepreneur at period 0 with the previous findings and the optimal behavior from date-1 onward is given by

$$\big[\alpha(\rho_1 - \rho_0 + \bar{x_1}^L) + (1-\alpha)C^H_{1,2}(\bar{x_1}^H)\big]i(\bar{x_1}^L, \bar{x_1}^H)$$

The sign of the first order condition of this objective function relative to $\bar{x_1}^L$ depends on the term $1 + \alpha\gamma_1^L - \alpha\pi - \alpha\rho_1$ which by Assumption 1 is negative. Thus, an entrepreneur never chooses to hoard liquidity for a boom period. This result is consistent with the fact that hoarding sacrifices investment scale and, with a boom, it provides no insurance since projects can be financed as they go. Note that even if any liquidity is hoarded, it is not used to reinvest but instead it is consumed. **What about $\bar{x_1}^H$?** During stress periods, a project cannot finance reinvestment as it goes. Thus, if it wants to survive, the banking entrepreneur needs to accumulate some liquidity. The F.O.C. of the payoff function with $\bar{x_1}^L = 0$ relative to $\bar{x_1}^H$ is given by

$$\big[\alpha(\rho_1 - \rho_0) + (1-\alpha)C^H_{1,2}(\bar{x_1}^H)\big]\frac{\partial}{\partial \bar{x_1}^H} i(\bar{x_1}^H) + (1-\alpha)i(\bar{x_1}^H)\frac{\partial}{\partial \bar{x_1}^H}C^H_{1,2}(\bar{x_1}^H)$$

The first term of this first order condition captures the cost of insuring as a fall in investment scale times the expected payoff while the second term captures the benefit from continuation. Not that this benefit is weighted by the probability that in fact a stress period happens given that any hoarding, even contingent, is wasted when the aggregate shock is a boom. I evaluate this F.O.C at two points given the discontinuity in $C^H_{1,2}(\bar{x_1}^H)$ (See above).

First, is it optimal to hoard any liquidity? To see this, I evaluate the F.O.C with respect to x_1^H at values close to zero. The sign of the derivative is given by the following term

$$1 - \pi - \alpha\big[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\big]$$

First note that it doesn't depend on x_1^H since it is a linear function. Therefore, the optimal choice is a corner solution. Now, when this term is negative, entrepreneurs do not hoard positive levels of liquidity.

B.2 Proposition 2 - Proof

To prove this proposition. I do so by contradiction. Define ω as stated, and the probability of a market stress is equal to z where $z \leq \omega$ by assumption. Suppose, now, that the economy is in a No Crisis Equilibrium, thus, by optimality, the FOC evaluated close to zero must be positive, Thus, the following relationship must hold.

$$0 < 1 - \pi - (1-z)\big[(\rho_0 - \gamma_1^L) + (1 - \frac{\rho_0}{\gamma_1^H})\big]$$

Using the fact that $1 - z$ is greater or equal to $1 - \omega$ which, by definition, is equal to $\frac{1-\pi}{(1-\frac{\rho_0}{\gamma_1^H})+(\rho_0-\gamma_1^L)}$, then

$$0 < 1-\pi-(1-z)\big[(\rho_0-\gamma_1^L)+(1-\frac{\rho_0}{\gamma_1^H})\big] \leq 1-\pi-\frac{1-\pi}{(1-\frac{\rho_0}{\gamma_1^H})+(\rho_0-\gamma_1^L)}\big[(\rho_0-\gamma_1^L)+(1-\frac{\rho_0}{\gamma_1^H})\big]$$

which is equal to zero. Then I found a contradiction.

B.3 Proposition 1 - Proof

In the case they choose to hoard liquidity, the question is how much. For this to happen, it must be true that $1-\pi \geq \alpha\big[(\rho_0-\gamma_1^L)+(1-\frac{\rho_0}{\gamma_1^H})\big]$. I now evaluate the F.O.C between $\bar{x_1}^H$ is between $1-\frac{\rho_0}{\gamma_1^H}$ and π, the sign of this F.O.C is given by

$$\gamma_1^H\big[1-\pi+\alpha\gamma_1^L+(1-\alpha)\big]-\rho_1-\alpha\rho_0\big[\gamma_1^H-1\big]$$

Note that, once again, this derivative is linear as it does not depend on x_1^H. Additionally, by Assumption 1, $\rho_1>\gamma_1^H\big[1-\pi+\gamma_L^L+(1-\alpha)\big]$, thus this derivative is negative. Consequently, an entrepreneur, if it chooses to hoard, it hoards $i[1-\frac{\rho_0}{\gamma_1^H}]$ which is the minimum amount necessary to continue at full-scale by complementing this liquidity with funding liquidity.

B.4 No Crisis Equilibrium - LFE

Whenever $1-\alpha$ is greater than ω and Assumptions 1 and 3 hold, the following characterizes the NO Crisis Equilibrium - LFE

- Date-2: Entrepreneurs' don't abscond
- Date-1: $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i\}$ and determined by strategy profile functions $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$
- Date-0: $\{c_0=0, x_A=0\}$ and $K_0=\{i=\frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)+(1-\alpha)(1-\frac{\rho_0}{\gamma_1^H})},\ M_0=A,\ \phi_0=i-A,\ d_f^L i=\pi i,\ d_f^H i=i\big[\pi-(1-\frac{\rho_0}{\gamma_1^H})\big],\ l_0^L=(\rho_0-\gamma_1^L)i,\ x_1^L=0, x_1^H=i(1-\frac{\rho_0}{\gamma_1^H})\}$ solve entrepreneurs problem at the initial period

Proof Optimal behavior at date-1 and date-2 was derived through backward induction. Given strategy profiles, $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$ and the previous discussion, since $1-\alpha$ is greater than ω, then the first order condition of the objective function is positive thus optimally choose a positive $x_1^H=$. Proposition 1 shows that it is optimal to select $x_1^H=i(1-\frac{\rho_0}{\gamma_1^H})$. ∎

B.5 Sudden Stop Equilibrium - LFE

Whenever $1-\alpha$ is less or equal than ω and Assumptions 1 and 3 hold, the following characterizes the Sudden Stop Equilibrium - LFE

- Date-2: Entrepreneurs' don't abscond
- Date-1: $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i\}$ and determined by strategy profile functions $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L=\frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H=\frac{l_1^H}{\gamma_1^H}$

- Date-0: $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)},\ M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = 0\ \}$ solve entrepreneurs problem at the initial period

Proof Optimal behavior at date-1 and date-2 was derived through backward induction. Given strategy profiles, $j^L,\ c_1^L,\ M_1^L,\ l_1^L,\ \phi_1^L = \frac{l_1^L}{\gamma_1^L},\ j^H,\ c_1^H,\ M_1^H,\ l_1^H,\ \phi_1^H = \frac{l_1^H}{\gamma_1^H}$ and the previous discussion, since $1-\alpha$ is less or equal to ω, then the first order condition of the objective function relative to x_1^H is negative thus an entrepreneur optimally choose $x_1^H = 0$. ∎

B.6 Banking Entrepreneurs Optimal Behavior with a LOLR

B.7 Period 1 - Optimal Behavior

In this section, I derive the best response functions when a LPP with funding cost $\hat{R}$ and fiscal capacity $\bar{\mu}$ is present.

I start at period $t = 1$ since contracts are designed for banking entrepreneurs to abide by requiring satisfying (14).

A given entrepreneur's period 1 optimization problem can be rewritten as follows:

$$\begin{aligned}
&\underset{\{c_1^s, M_1^s, \phi_1^s, \tau\}}{\text{Maximize}}\ \big[1-\gamma_1^s\big]c_1^s + \big[\rho_1 - \gamma_1^s\big]j^s + \gamma_1^s x_1^s + \big[\gamma_1^s - \hat{R}\big]\tau \\
&\text{subject to: } j^s = min\{\frac{M_1^s + \tau^s}{1-\phi_1^s}, i\} \\
&\qquad \hat{R}(1-\bar{\mu})\tau + \big[\gamma_1^s\phi_1^s - \rho_0\big]j^s \le 0 \\
&\qquad c_1^s + M_1^s \le x_1^s \\
&\qquad \{c_1^s, M_1^s, \phi_1^s, \tau\}\ non-negative, \text{ given } \{x_1^s,\ i,\ \hat{R},\ \bar{\mu}\}
\end{aligned}$$

I start by solving this problem for a boom First, consider when $\hat{R} \ge \gamma_1^L$. For a given $\{x_1^s,\ i,\ \hat{R} \ge \gamma_1^L,\ \bar{\mu}\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. Moreover, note, from the objective function, that τ should be as small as possible since $\hat{R} \ge \gamma_1^L$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 1, \tau = 0\}$ is a optimal answer with payoff $\big[\rho_1 - \gamma_1^L\big]i + x_1^L$. To see this, first, note that this solution is feasible since $\gamma_1^L i < \rho_0 i$. I show optimality by contradiction. Suppose there exists $\{\hat{c_1^s}, \hat{M_1^s}, \hat{\phi_1^s}\}$ that is feasible and produces a strictly greater payoff. This is not possible because $1 > \gamma_1^L$ in this scenario, $\hat{j^L} \le i$ and $\hat{C_1^L} \le x_1^L$ due to feasibility, $\gamma_1^L < \rho_1$, and $\gamma_1^L < \hat{R}$ as an assumption for this scenario. **Now, consider when $\hat{R} < \gamma_1^L$.** For a given $\{x_1^s,\ i,\ \hat{R} < \gamma_1^L,\ \bar{\mu}\}$, recall that $\gamma_1^L < \rho_0$ which implies that $\gamma_1^L < 1$. Moreover, note, from the objective function, that τ should be as large as possible since $\hat{R} \ge \gamma_1^L$. I argue that, in this scenario, $\{j^L = i, c_1^L = x_1^L, M_1^L = 0, \phi_1^L = 0, \tau = i\}$ is a optimal answer with payoff $\big[\rho_1 - \hat{R}\big]i + x_1^L$. Note that this candidate solution is feasible for any $\bar{\mu}$ since $\rho_0 > \gamma_1^L > \hat{R} \rightarrow \rho_0 > \hat{R}(1-\bar{\mu})$. I prove optimality by contradiction. I assume that there exists another feasible candidate that generates a greater payoff than $\big[\rho_1 - \hat{R}\big]i + x_1^L$. This is not possible because any feasible τ is bounded from above by i, the same as any feasible j^L, while any feasible c_1^L is bounded by x_1^L. Below the optimal strategies for an entrepreneur during a boom. To differentiate from optimal behavior in LFE, I denote these functions with a tilde.

For all $x_1^L \geq 0$

$$\tilde{j}^L = \begin{cases} i & \text{for all } \hat{R} \end{cases}$$

$$\tilde{c}_1^L = \begin{cases} x_1^L & \text{for all } \hat{R} \end{cases}$$

$$\tilde{M}_1^L = \begin{cases} 0 & \text{for all } \hat{R} \end{cases}$$

$$\tilde{l}_1^L = \begin{cases} \gamma_1^L i & \text{if } \hat{R} \geq \gamma_1^L \\ 0 & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\rho_0 \tilde{j}^L - \tilde{l}_1^L - (1-\bar{\mu})\hat{R}\tau = \begin{cases} \left[\rho_0 - \gamma_1^L\right] i & \text{if } \hat{R} \geq \gamma_1^L \\ \left[\rho_0 - (1-\bar{\mu})\hat{R}\right] i & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\tilde{\tau}^L = \begin{cases} 0 & \text{if } \hat{R} \geq \gamma_1^L \\ i & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

$$\tilde{C}_{1,2}^L = \begin{cases} \left[\rho_1 - \gamma_1^L\right] i + x_1^L & \text{if } \hat{R} \geq \gamma_1^L \\ \left[\rho_1 - \hat{R}\right] i + x_1^L & \text{if } \hat{R} < \gamma_1^L \end{cases}$$

I continue by solving this problem for a stress period For a given $\{x_1^H, i, \hat{R}, \bar{\mu}\}$, recall that $\gamma_1^H > \rho_0$ which implies that $\phi_1^H = 1$ is no longer possible (no *finance as you go*). Moreover, $\gamma_1^H > 1$ which implies that it is better to lend at international markets than to consume, thus, c_1^H is equal to zero.

By assumptions, I only consider scenarios where $0 < \hat{R} \leq \gamma_1^H$. In this case, τ has to be the largest possible given that it is not, in any case, more expensive than market and funding liquidity. This is reflected in the objective function. Simultaneously, τ's cost on pledgeable income is *smoother* due to fiscal capacity $(1-\bar{\mu})$, so, even if $\hat{R} = \gamma_1^H$ and for any given level of x_1^H, an entrepreneur can borrow from the LOLR as a minimum (if $\bar{u} = 0$) the same amount as in international markets. This suggests that is weakly optimal to exhaust pledgeable income with τ. To see this more formally, **I first consider when** $(1-\bar{\mu})\hat{R} \leq \rho_0$ That is, for a given $\hat{R}$, the level of fiscal capacity is such that a project can finance as it goes using public funding liquidity. I argue that $\{j^H = i, c_1^H = 0, M_1^H = 0, \phi_1^H = 0, \tau = i\}$ is optimal with payoff $\left[\rho_1 - \hat{R}\right] i + \gamma_1^H x_1^H$. Note that this candidate is feasible since j^H is equal to i and $\tau = i$ is possible since $(1-\bar{\mu})\hat{R} \leq \rho_0$. I prove optimality by contradiction. I suppose that there is another feasible solution with a payoff greater than $\left[\rho_1 - \hat{R}\right] i + \gamma_1^H x_1^H$. However, this is not possible because γ_1^H is greater than 1, and any feasible τ and j are bounded by i. Note that, since there is an upper limit on $\hat{R}$ equal to γ_1^H, then the case of $(1-\bar{\mu})\hat{R} \leq \rho_0$ is the only possible scenario for when $\bar{\mu} \geq 1 - \frac{\rho_0}{\gamma_1^H}$. Next, consider **when** $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$. In this scenario, projects can no longer finance as they go using public funding, they need to complement with market liquidity. I argue that when $x_1^H \geq i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$, $\{j^H = i, c_1^H = 0, M_1^H = i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big], \phi_1^H = 0, \tau = \frac{\rho_0}{(1-\bar{\mu})\hat{R}}i\}$ is optimal with payoff $\big[\rho_1-\frac{\rho_0}{1-\bar{\mu}}\big]i+\gamma_1^H\big[x_1^H-i(1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}})\big]$. First, note that this candidate is feasible precisely because $\rho_0 < (1-\bar{\mu})\hat{R} < \hat{R} \leq \gamma_1^H$ and $x_1^H \geq i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$. I show optimality by contradiction. There is no other feasible candidate that creates a greater payoff since any j^H is bounded by i, and, in this scenario, any feasible τ is bounded by $\frac{\rho_0}{(1-\bar{\mu})\hat{R}}i$. **Now, I turn to when** $x_1^H < i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$, where I argue that $\{j^H = \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}, c_1^H = 0, M_1^H = x_1^H, \phi_1^H = 0, \tau = \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}\}$ is optimal with payoff $\big[\rho_1-\frac{\rho_0}{1-\bar{\mu}}\big]\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$. This candidate is feasible. To see this, $j^s < i$ since $x_1^H < i\big[1-\frac{\rho}{(1-\bar{\mu})\hat{R}}\big]$ Also, the incentive compatibility constraint is binding in this scenario, so it is feasible, and, finally $c_1 + M_1$ is equal to x_1^H which, by definition, is less or equal to x_1^H. Once again, I show optimality by contradiction. There is another solution that generates a greater payoff. This is not possible since: i) any feasible j^H is bounded by $\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$ since $x_1\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$, $\hat{R}(1-\bar{\mu}) \leq \gamma_1^H$, and any ϕ_1^H, τ are non-negative, ii) any feasible τ are argued earlier is bounded by $\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}}$ and $\gamma_1^H \geq \hat{R}$.

Below the optimal strategies for an entrepreneur during a stress episode. To differentiate from optimal behavior in LFE, I denote these functions with a tilde. I also define $\bar{\mu}_{AE}$ equal to $1-\frac{\rho_0}{\gamma_1^H}$

$$\tilde{j}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \\ i & \quad \text{for all } x_1^H \geq i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \end{cases}$$

$$\tilde{c}_1^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H < i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \\ 0 & \quad \text{for all } x_1^H \geq i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \end{cases}$$

$$\tilde{M}_1^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ 0 & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ x_1^H & \quad \text{for all } x_1^H < i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \\ i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] & \quad \text{for all } x_1^H \geq i\big[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] \end{cases}$$

$$\tilde{l}_1^H + \hat{R}(1-\bar{\mu})\tau = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ (1-\bar{\mu})\hat{R}i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \rho_0 \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ \rho_0 i & \quad \text{for all } x_1^H \geq i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{\tau}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ i & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \frac{\rho_0}{(1-\bar{\mu})\hat{R}} \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ \frac{\rho_0}{(1-\bar{\mu})\hat{R}} i & \quad \text{for all } x_1^H \geq i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

$$\tilde{C}_{1,2}^H = \begin{cases} & \text{if } \hat{R}(1-\bar{\mu}) \leq \rho_0 \mid \bar{\mu} \geq \bar{\mu}_{AE} \\ \left[\rho_1 - \hat{R}\right]i + \gamma_1^H x_1^H & \quad \text{for all } x_1^H \geq 0 \\ & \text{if } \gamma_1^H \geq \hat{R}(1-\bar{\mu}) > \rho_0 \ \& \ \bar{\mu} < \bar{\mu}_{AE} \\ \left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right] \frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}} & \quad \text{for all } x_1^H < i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \\ \left[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\right]i + \gamma_1^H\left[x_1^H - i(1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}})\right] & \quad \text{for all } x_1^H \geq i\left[1-\frac{\rho_0}{(1-\bar{\mu})\hat{R}}\right] \end{cases}$$

B.8 Period 0 - Optimal Behavior

Recall that a given entrepreneur's period 0 optimization problem is as follows:

$$\begin{aligned} \underset{\{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\}}{\text{Maximize}} \quad & c_0 + \alpha\left[C_{1,2}^L - l_0^L\right] + (1-\alpha)\left[C_{1,2}^H\right] \\ \text{subject to: } & \alpha\left[l_0^L + d_f^L i\right](1-\alpha)\left[d_f^H i\right] = i - M_0 \\ & d_f^s i + d_e^s i = \pi i \\ & l_0^L \ \epsilon \left[0, \rho_0 j^L - l_1^L\right] \\ & A - F_0 - c_0 - M_0 = x_A \\ & d_e^s i + x_A = x_1^s \\ & \{c_0,\ x_A,\ i,\ M_0,\ d_f^L i,\ d_f^H i,\ l_0^L\} \text{ non-negative, given the optimal} \\ & \text{functions at date-1 that depend on } \{x_1^L, x_1^H\} \end{aligned}$$

The proof that any entrepreneur chooses $C_0 = 0, x_1^L = 0, l_0^L = \left[\rho_0 - \gamma_1^L\right]i$ follows the same steps as in the Laissez Faire Equilibrium. If interested, I refer the reader to it. Similarly, regarding M_0, incentives for entrepreneurs are to invest in the project all of their net worth as in the LFE. However, with an LOLR, entrepreneurs can only invest their disposable net worth $A - F_0 = M_0$.

What about $\bar{x_1}^H$? The decision on how much to hoard at $t = 0$ will depend on the expectation of $\hat{R}$ and the LOLR's fiscal capacity. **First, I consider a pair $\{\hat{R}, \bar{\mu}\}$ such that** $\hat{R}(1-\bar{\mu}) = \rho_0$. In this scenario, the objective function is given by

$$\Big[\alpha\big[\rho_1 - \rho_0\big] + (1-\alpha)\big[\rho_1 - \hat{R} + \gamma_1^H \bar{x}_1^H\big]\Big]\frac{A - F_0}{1 - \pi - \alpha(\rho_0 - \gamma_1^L) + (1-\alpha)\bar{x}_1^H}$$

where the sign of the derivative is determined by

$$\gamma_1^H\big[1 - \pi - \alpha(\rho_0 - \gamma_1^1)\big] - \alpha(\rho_1 - \rho_0) - (1-\alpha)(\rho_1 - \hat{R})$$

which, because $\gamma_1^H \geq \hat{R}$, is less or equal to

$$\gamma_1^H\big[1 - \pi + \alpha\gamma_1^L + (1-\alpha)\big] - \rho_1 - \alpha\rho_0(\gamma_1^H - 1)$$

This last term is strictly negative because of Assumption 1. Note that this is true independently of probabilities of events. **Next, I consider a pair $\{\hat{R}, \bar{\mu}\}$ such that** $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$ If an entrepreneur's expectations are within this environment, then the sign of derivative of the objective function with respect to $\bar{x_1^H}$ evaluated close to zero depends on the sign of

$$\Big[(\rho_1 - \frac{\rho_0}{1-\bar{\mu}})(1 - \pi - \alpha(\rho_0 - \gamma_L^L)) - \alpha\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big](\rho_1 - \rho_0)\Big]$$

This term can be negative or positive depending on α. I define $\omega(\bar{\mu}, \hat{R})$ equal to

$$\frac{(\rho_1 - \rho_0)\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] + \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big]\big[(\rho_0 - \gamma_1^L) - (1-\pi)\big]}{(\rho_1 - \rho_0)\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big] + \big[\rho_1 - \frac{\rho_0}{1-\bar{\mu}}\big](\rho_0 - \gamma_1^L)} \tag{27}$$

It is quite straight forward to show that if $(1-\alpha) \leq \omega(\bar{\mu}, \hat{R})$, and expectations over pair $\{\hat{R}, \bar{\mu}\}$ are such that $\rho_0 < (1-\bar{\mu})\hat{R} \leq \gamma_1^H$, then an entrepreneur's optimal answer is $x_1^H = 0$. Likewise, with same expectations but when $(1-\alpha) > \omega(\bar{\mu}, \hat{R})$ then x_1^H is equal to $i\big[1 - \frac{\rho_0}{(1-\bar{\mu})\hat{R}}\big]$.

B.9 Mature Fiscal Capacity Equilibrium

Whenever $\bar{\mu} \ \epsilon \ \big[\mu_A, 1\big]$, and Assumptions 1 and 3 hold, the following Mature Fiscal Capacity Equilibrium Exists -

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR's collects $\gamma_1^H i$ and uses it to redeem bonds to foreign lenders.
- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L$, $\tilde{c}_1^L$, $\tilde{M}_1^L$, $\tilde{l}_1^L$, $\tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L}$, τ^L, $\tilde{j}^H$, $\tilde{c}_1^H$, $\tilde{M}_1^H$, $\tilde{l}_1^H$, $\tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H}$, τ^H. Given τ^L and τ^L, the LOLR issues $B_1 = i$

during market stress and zero during a boom. At this point, $\{j^L, j^H\}$ are equal to $\{i, i\}$

- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)},\ M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = 0\}$ solve entrepreneurs problem at the initial period. Given $\{x_1^L, x_1^H\}$, the LOLR chooses optimally $F_0 = 0$

Proof Choose $\bar{\mu} \geq \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. Suppose they are not, then $0 < \gamma_1^H B_1 - \hat{R}\tau \rightarrow \gamma_1^H i - \gamma_1^H i = 0 \rightarrow 0 < 0$ which is a contradiction. This only holds if an LOLR can collect fully $\hat{R}\tau$. Again, suppose that is not possible. Then, it must be true that $0 < (1-\bar{\mu})\hat{R}\tau - \rho_0 i = (1-\bar{\mu})\gamma_1^H i - \rho_0 i$ which is strictly less than zero because $\bar{\mu} \geq \mu_A$ and hence a contradiction . This result is also consistent with entrepreneurs not absconding. At date-1, given $\hat{R} = \gamma_1^S$ and $\bar{\mu} \geq \mu_A$, then $\gamma_1^H(1-\bar{\mu}) \leq \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = i$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = i$ of course. Given $\hat{R} = \gamma_1^H$ and $\bar{\mu} \geq \mu_A$, it is optimal for entrepreneurs to choose $x_1^H = 0$ by Assumption 1. Now, is it optimal for the LOLR given $x_1^H = 0$ and $\bar{\mu} \geq \mu_A$ to choose F_0 equal to zero. Suppose it is not. Then there is a feasible $\hat{F}_0$ greater than zero that generates a lower welfare cost, Then it must hold that $0 > \psi\big[(\hat{F}_0 - F_0)\big]\kappa(0) + (1-\alpha)(L(i) - L(i)) = \psi\big[(\hat{F}_0)\big]$ which is strictly greater than zero since any feasible $\hat{F}_0$ is non negative and by assumption $\hat{F}_0 > 0$ ∎

B.10 Sudden Stop Equilibrium - No Reserves

Whenever $\bar{\mu} < \mu_A$, $(1-\alpha) < \omega(\gamma_1 H, bar\mu)$, and Assumptions 1 and 3 hold, the following following Sudden Stop Equilibrium exists

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR doesn't collect since there are no outstanding bonds. Note that, after a stress, entrepreneurs don't consume either because projects were shutdown.

- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L,\ \tilde{c}_1^L,\ \tilde{M}_1^L,\ \tilde{l}_1^L,\ \tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L},\ \tau^L,\ \tilde{j}^H,\ \tilde{c}_1^H,\ \tilde{M}_1^H,\ \tilde{l}_1^H,\ \tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H},\ \tau^H$. $\{\tau^L, \tau^H\}$ are equal to $\{0, 0\}$ and doesn't need to issue any bonds in either state. At this point, $\{j^L, j^H\}$ are equal to $\{i, 0\}$

- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)},\ M_0 = A,\ \phi_0 = i - A,\ d_f^L i = \pi i,\ d_f^H i = \pi i,\ l_0^L = (\rho_0 - \gamma_1^L)i,\ x_1^L = 0, x_1^H = 0\}$ solve entrepreneurs problem at the initial period. I assume that the LOLR cannot accumulate reserves.

Proof Choose $\bar{\mu} < \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. This is obvious since there are no outstanding. Since projects shutdown, entrepreneurs don't have any incentives to abscond after a stress period. At date-1, given $\hat{R} = \gamma_1^S$, $\bar{\mu} < \mu_A$, then $\gamma_1^H(1-\bar{\mu}) > \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\frac{x_1^H}{1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}}$ which is equal to zero since $x_1^H = 0$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = 0$ of course. Given $\hat{R} = \gamma_1^H$ and $\bar{\mu} < \mu_A$, and, that $(1-\alpha) < \omega(\gamma_1 H, bar\mu)$ it is optimal for entrepreneurs to choose $x_1^H = 0$ since the probability of a market stress is lower than the threshold at which the FOC of entrepreneurs shifts to positive. Now, is it optimal for the LOLR given $x_1^H = 0$ and $\bar{\mu} < \mu_A$ to choose F_0 equal to zero? Probably not Suppose it is not. Then there is a feasible $\hat{F}_0$ greater than zero that generates a lower welfare cost, Then it must hold that $0 > \psi\big[(\hat{F}_0 - F_0)\big]\kappa(0) + (1-\alpha)(L(i) - L(i)) = \psi\big[(\hat{F}_0)\big]$ which is strictly greater than zero since any feasible $\hat{F}_0$ is non negative and by assumption $\hat{F}_0 > 0$ ∎

B.11 No Crisis Equilibrium - No Reserves

Whenever $\bar{\mu} < \mu_A$, $(1-\alpha) > \omega(\gamma_1 H, bar\mu)$, and Assumptions 1 and 3 hold, the following No Crisis Equilibrium - No Reserves Exists -

- Date-2: Entrepreneurs' don't abscond, and consume according to $\{\tilde{c}_2^L, \tilde{H}_2^L\}$. After a boom event, an LOLR has no action. Following a market stress, LOLR's collects $\gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i$ and uses it to redeem bonds to foreign lenders.

- Date-1: Given the realized shock, the LOLR sets $\hat{R}$ equal to γ_1^S accordingly. $\{c_1^s, K_1^s\}_{L,H}$ are contingent on $\{x_1^L, x_1^H, i, \hat{R}\}$ and determined by strategy profile functions $\tilde{j}^L$, $\tilde{c}_1^L$, $\tilde{M}_1^L$, $\tilde{l}_1^L$, $\tilde{\phi}_1^L = \frac{l_1^L}{\gamma_1^L}$, τ^L, $\tilde{j}^H$, $\tilde{c}_1^H$, $\tilde{M}_1^H$, $\tilde{l}_1^H$, $\tilde{\phi}_1^H = \frac{l_1^H}{\gamma_1^H}$, τ^H. Given τ^L and τ^L, the LOLR issues $B_1 = \big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i$ during market stress and zero during a boom. At this point, $\{j^L, j^H\}$ are equal to $\{i, i\}$

- Date-0: Given $\bar{\mu}, \gamma_1^S\}$, $\{c_0 = 0, x_A = 0\}$ and $K_0 = \{i = \frac{A}{1-\pi-\alpha(\rho_0-\gamma_1^L)+(1-\alpha\bar{x}_1^H}$, $M_0 = A$, $\phi_0 = i - A$, $d_f^L i = \pi i$, $d_f^H i = \pi i - \big[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i$, $l_0^L = (\rho_0 - \gamma_1^L)i$, $x_1^L = 0, x_1^H = i[1 - \gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]]\}$ solve entrepreneurs problem at the initial period. I assume that the LOLR cannot collect Reserves.

Proof Choose $\bar{\mu} < \mu_A$. Let start by showing that bonds are redeemable after a market stress with no reserves. Suppose they are not, then $0 < \gamma_1^H B_1 - \hat{R}\tau \to \gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i - \gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i = 0 \to 0 < 0$ which is a contradiction. This only holds if an LOLR can collect fully $\hat{R}\tau$. Again, suppose that is not possible. Then, it must be true that $0 < (1-\bar{\mu})\hat{R}\tau - \rho_0 i = (1-\bar{\mu})\gamma_1^H\big[\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]i - \rho_0 i$ which is strictly less than zero by simplification and hence a contradiction . This result is also consistent with entrepreneurs not absconding. At date-1, given $\hat{R} = \gamma_1^S$ and $\bar{\mu} < \mu_A$, then $\gamma_1^H(1-\bar{\mu}) > \rho_0$, so, entrepreneurs demand $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}i$. Given that $F_0 = 0$, then $\hat{R}$ is set equal to γ_1^S for any demand τ, including $\tau^L = 0$ and $\tau^H = \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}i$ of course. Given $\hat{R} = \gamma_1^H$, $\bar{\mu} < \mu_A$, and that $(1-\alpha) \geq \omega(\gamma_1^H, \bar{\mu})$ it is optimal for entrepreneurs to choose $x_1^H = i[1 - \frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}]$ since the probability of a market stress is more than enough to compensate for the sacrifice in investment scale. Note that when $(1-\alpha) \geq \omega(\gamma_1^H, \bar{\mu})$ ∎

B.12 Lender of Last Resort Optimal Behavior

At $t = 2$, the LOLR collects any claims on entrepreneurs to redeem bonds potentially issued at $t = 1$. The LOLR collects a total of $\hat{R}\tau$ where a share $\bar{\mu}$ comes from entrepreneurs directly and the remainder from projects as long as entrepreneurs don't abscond. If entrepreneurs abscond, then an LOLR cannot claim $(1 - \bar{\mu})\hat{R}\tau$. As a result, total revenue is limited to $\bar{\mu}\hat{R}\tau$. However, by design, Contracts K_1^S are such that entrepreneurs don't abscond so it is fair to say that total revenue can be collected. Thus, for bonds to be redeemable, the following condition must hold

$$\gamma_1^S B_1 \leq \hat{R}\tau$$

At $t = 1$, the LOLR defines $\hat{R}$, while it issues B_1, and depletes f_1 of their reserves stock (F_0) to cover any demand for public liquidity. So, given τ, B_1 is equal to the $max\{0, \tau - f_1\}$. By replacing this in the previous condition and the fact that $f_1 \leq F_0$, then

$$\gamma_1^s \tau \leq \hat{R}\tau + F_0$$

Then, this condition with equality and knowing that $\hat{R}$ is non-negative, it is straight forwad to derive (18). Note that as long as is greater or equal than (18), (17) is satisfied. To see this, assume that there is exists a R such that is greater or equal to $\bar{R}(\tau, F_0)$ and it doesn't satisfy (17). If this is true then the following must hold, for positive τ,

$$0 < \gamma_1^S \tau - R\tau - \gamma_1^S F_0 \leq \gamma_1^S \tau - \bar{R}\tau - \gamma_1^S F_0 = \gamma_1^S \tau - \gamma_1^S \big[1 - \frac{F_0}{\tau}\big]\tau - \gamma_1^S F_0$$

However, this is a contradiction since the last term is equal to zero. Thus, there is no such R. This condition is also satisfied for τ equal to zero since zero is equal to the product of γ_1^s times zero. This proves that setting $\hat{R}$ equal to $\bar{R}(\tau, F_0)$ guarantees that (17) is satisfied. Thus, at $t = 1$, the LOLR with a given stock of reserves F_0 observes τ and sets $\hat{R}$ accordingly. One important clarification is what should a LOLR do with F_0 is τ is equal to zero. Since there is no demand, then f_1 and B_1 is zero by definition. The LOLR has two options with F_0, either lend it at international markets and rebate the return to entrepreneurs at $t = 2$, or rebate it immediately to entrepreneurs at $t = 1$. Since the LOLR has no preference over entrepreneurs consumption, I establish that it rebates everything at the end of $t = 2$ which is consistent with what determines ψ.

The optimal behavior at $t = 0$ is, given x_1^H, to choose $F_0 \leq A$ to minimize

$$\psi F_0 \kappa(x_1^H) + (1 - \alpha)L(\tilde{j}^H)$$

Note that this version of the objective function already includes that entrepreneurs don't hoard liquidity for booms and that they are able to reinvest at full-scale, even without LOLR assistance. The expected welfare cost objective function has a lower bound at zero when F_0 is equal to zero and $j^H = 1$. Additionally, $\bar{R}(\tau, 0)$ is equal to γ_1^H for any non-negative τ. **First, I consider the case of Mature LOLR** Choose $\bar{\mu} \geq \mu_A$. I argue that for any γ_1^H, the optimal response is F_0 equal to zero. The reason is that when $\bar{\mu} \geq \mu_A$, then $(1 - \bar{\mu})\gamma_1^H \leq \rho_0$. In this scenario, as determined by $\tilde{j}^H$, projects continue at full scale regardless of x_1^H. To prove this, suppose that there exists a positive $\tilde{F}_0$ such that is generates a lower payoff than F_0. This is not possible because $\tilde{F}_0$ incurs in the opportunity cost $\psi\tilde{F}_0\kappa(x_1^H)$ while not reducing the welfare losses due to partial liquidation since, even with F_0 equal to zero, $\tilde{j}^H$ is equal to i. **Now, I consider the case of an economy with a LOLR with $\bar{\mu} < \mu_A$.** I argue that if $x_1^H \geq i[1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}$ then the optimal F_0 is equal to zero. Similar to the previous argument, with this amount of liquidity, entrepreneurs are able to continue at full scale at a cost of public liquidity equal to γ_1^H which results from $F_0 = 0$. Thus, F_0 is optimal because it reaches the lower bound of the expected welfare function. Any positive F_0 incurs in cost $\psi\kappa(x_1^H)$ but cannot reduce the welfare losses beyond zero. Next, I consider the case when x_1^H is equal to zero. With no market liquidity, projects are forced to shutdown ($\tilde{j}^H = 0$) unless the marginal cost of public liquidity is at the most such that $(1-\bar{\mu})R = \rho_0$. This lowers the level of $\hat{R}$ sufficiently such that any unit borrowed from LOLR increases pledgeable income in the same magnitude. This is the condition for example, for a cost of funding liquidity to be such that projects can be self-financed. By rearranging (18), you find F_0 as a function of $\hat{R}$.

$$F(\hat{R},\bar{\mu}) = \big[1-\frac{\hat{R}}{\gamma_1^H}\big]\tau(\hat{R},\bar{\mu})$$

Function $(\hat{R},\bar{\mu})$ is not completely determined since the demand of public liquidity is a function of $\hat{R}$ and F_0 it self. However, this relationship can be used to find $\bar{F}(\bar{\mu})$ which is the level of reserves such that $(1-\bar{\mu})\hat{R} = \rho_0$ when $x_1^H = 0$

$$\bar{F}(\bar{\mu}, x_1^H) = A\kappa(0)\frac{1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}}{1+\big[1-\frac{\rho_0}{(1-\bar{\mu})\gamma_1^H}\big]\kappa(0)}$$

It is worth pointing out that $\bar{F}(\bar{\mu})$ is decreasing and strictly concave with respect to $\bar{\mu}$. Thus if a LOLR accumulates $\bar{F}(\bar{\mu})$, then it incurs in a welfare cost equal to $\psi\bar{F}(\bar{\mu})\kappa(0)$. I argue that this is optimal if the expected welfare cost of shutdown is too large. To see this, first, consider a F_0 that is greater than $\bar{F}(\bar{\mu})$ but generates a lower welfare cost. This is not possible since $\bar{F}(\bar{\mu})$ is by definition the minimum amount of reserves that achieve full scale reinvestment when x_1^H is zero. Now consider an F_0 that is lower than $\bar{F}(\bar{\mu})$. Recall that with $x_1^H = 0$, entrepreneurs cannot reinvest at all, and, thus, shutdown their projects. So, $j^H = 0$ for all $F_0 < \bar{F}(\bar{\mu})$ Among those F_0 lower than $\bar{F}(\bar{\mu})$, F_0 equal to zero generates the lower welfare costs since projects shutdown but it doesn't incur in the opportunity cost of deviating resources. Thus, I define set $\Lambda(\bar{\mu}) = \{z \mid z \leq \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})\}$ I argue that when $1-\alpha$ belongs to $\Lambda(\bar{\mu})$, then $F_0 = 0$ is optimal. To see this, suppose that it is not. Therefore, I assume that $\bar{F}(\bar{\mu})$ is optimal then $0 > \psi\kappa(0)(\bar{F}(\bar{\mu})-0)+(1-\alpha)(0-L(0))$ which simplified is $0 > \psi\kappa(0)\bar{F}(\bar{\mu})-(1-\alpha)L(0)) \geq \psi\kappa(0)\bar{F}(\bar{\mu}) - \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})L(0))$ since $(1-\alpha)$ belongs to $\Lambda(\bar{\mu})$. Note that the last term is equal to zero, thus, I get a contradiction and $F_0 = 0$ is optimal. Similarly, when $1-\alpha$ doesn't belong to $\Lambda(\bar{\mu})$, then $\bar{F}(\bar{\mu})$ is optimal. To see this suppose that it is not. Then, if F_0 is optimal then it mus be true that $0 > -\psi\kappa(0)\bar{F}(\bar{\mu}) + (1-\alpha)L(0)) \geq \psi\kappa(0)\bar{F}(\bar{\mu}) + \frac{\psi\kappa(0)}{L(0)}\bar{F}(\bar{\mu})L(0))$ since $(1-\alpha)$ doesn't belong to $\Lambda(\bar{\mu})$. Again, note that the last term is equal to zero, thus, I get a contradiction and $F_0 = \bar{F}(\bar{\mu})$ is optimal. A comment is relevant. There is the possibility that set $\Lambda(\bar{\mu})$ is empty for feasible probability of market stress if $L(0)$ is too high. Finally, I consider the case when x_1^H is strictly between zero and

$i\big[1 - \frac{\rho}{(1\bar{\mu})\gamma_1^H}\big]$. In this scenario, entrepreneurs have positive levels of liquidity such that the optimal F_0 is an solution determined by the first order condition

$$\psi\kappa(x_1^H) + (1-\alpha)\frac{\partial L(j^s)}{\partial j^s}\frac{\partial j^s}{\partial \hat{R}}\frac{\partial \hat{R}}{\partial F_0}$$

In this scenario, given the continuity of functions, it is possible that the LOLR will accept some partial liquidation in order to reduce the cost of hoarding reserves.

B.13 Proof of Proposition 9

I focus on equilibria with non-defaultable debt. Thus, aggregate external liabilities $(\phi_1 j + B_1)$ cannot be greater than what the economy produces $(\frac{\rho_1}{\gamma_1^H}j)$. First, I assume that the definition of $\bar{B}_1$ is true. In equilibrium, $\phi_1 j + B_1$ cannot be greater than $\frac{\rho_0}{\gamma_1^H}j + \bar{\mu}B_1$ due to the incentive compatibility constraint. Given the definition of $\bar{B}_1$, $\frac{\rho_1}{\gamma_1^H}j$ cannot be greater than $\frac{\rho_0}{\gamma_1^H}j + \bar{\mu}\bar{T}(\mu)\frac{\rho_1}{\gamma_1^H}j$. Now, I show that B_1 in equilibrium cannot be greater than $\bar{B}_1$. Let $\mu \geq \theta$ then $\bar{B}_1 = \frac{\rho_1}{\gamma_1^H}j$. Thus, any $B_1 \leq \frac{\rho_1}{\gamma_1^H}j$ must hold because of limited liability and non-defaultable debt. Let $\mu < \theta$, then $\bar{B}_1 = \frac{\rho_0}{(1-\mu)\gamma_1^H}j$. Since the incentive compatibility constraint holds in equilibrium, then B_1 has to be less or equal $\frac{\rho_0}{(1-\mu)\gamma_1^H}j$. ∎

B.14 Proof of Proposition 10

To prove this proposition, it suffices to show that $min\{\frac{\rho_0}{\gamma_1^H}j + \mu_{CO}\bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H}j,\ \frac{\rho_1}{\gamma_1^H}j\}$ is greater or equal to j, or, equivalently, that $min\{\frac{\rho_0}{\gamma_1^H} + \mu_{CO}\bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\}$ is greater or equal to 1. This condition implies that only external liabilities are enough to cover full-scale reinvestment. **I start by proving that $\mu_{CO} = 1$ and $\mu_T \geq \bar{\mu}_T$ is true by contradiction.** I assume that $\mu_{CO} = 1$ and $\mu_T \geq \bar{\mu}_T$ holds but that $min\{\frac{\rho_0}{\gamma_1^H} + \mu_{CO}\bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} < 1$. Let $\mu_T \geq \theta$ then we have that

$$0 < 1 - min\{\frac{\rho_0}{\gamma_1^H} + \bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} = 1 - \frac{\rho_1}{\gamma_1^H}$$

which is not true by Assumption 1. Let $\mu_T < \theta$ then we have that

$$0 < 1 - min\{\frac{\rho_0}{\gamma_1^H} + \bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} = 1 - \frac{\rho_0}{\gamma_1^H} - \frac{\rho_0}{(1-\mu_T)\gamma_1^H} = \frac{1}{1-\mu_T}\Big[\left(1 - 2\frac{\rho_0}{\gamma_1^H}\right) - \mu_T\left(1 - \frac{\rho_0}{\gamma_1^H}\right)\Big]$$

Since $\mu_T \geq \bar{\mu}_T$ then

$$0 < \frac{1}{1-\mu_T}\Big[\left(1 - 2\frac{\rho_0}{\gamma_1^H}\right) - \bar{\mu}_T\left(1 - \frac{\rho_0}{\gamma_1^H}\right)\Big]$$

Replacing the definition of $\bar{\mu}_T$, one can see that the difference inside the brackets is equal to zero, so we have a contradiction $(0 < 0)$. Now, **I move to prove that $\mu_T = 1$ and $\mu_{CO} \geq \bar{\mu}_{CO}$ is true by contradiction.** I assume that $\mu_T = 1$ and $\mu_{CO} \geq \bar{\mu}_{CO}$ holds but that $min\{\frac{\rho_0}{\gamma_1^H} + \mu_{CO}\bar{T}(\mu_T)\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} < 1$. Let $\mu_{CO} \geq \theta$, then replacing the assumptions we have that

$$0 < 1 - min\{\frac{\rho_0}{\gamma_1^H} + \mu_{CO}\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} = 1 - \frac{\rho_1}{\gamma_1^H}$$

which is not true by Assumption 1. Let $\mu_{CO} < \theta$ then we have that

$$0 < 1 - min\{\frac{\rho_0}{\gamma_1^H} + \mu_{CO}\frac{\rho_1}{\gamma_1^H},\ \frac{\rho_1}{\gamma_1^H}\} = 1 - \frac{\rho_0}{\gamma_1^H} - \mu_{CO}\frac{\rho_1}{\gamma_1^H}$$

Since $\mu_{CO} \geq \bar{\mu}_{CO}$ then

$$0 < 1 - \frac{\rho_0}{\gamma_1^H} - \bar{\mu}_{CO}\frac{\rho_1}{\gamma_1^H} = 0$$

where the last equality results from replacing the definition of $\bar{\mu}_{CO}$, and solving the difference. So, we have a contradiction ($0 < 0$). ∎

B.15 Proof of Corollary 6

As long as Assumptions 1 and 3 hold, then $\mu_A > 0$. Since $\bar{\mu}_{CO} = \frac{\gamma_1^H}{\rho_1}\mu_A$ and $\gamma_1^H < \rho_1$ by Assumption 1 then $\mu_A > \bar{\mu}_{CO} > 0$ for any feasible γ_1^H. Note that μ_A, $\bar{\mu}_{CO}$, and $\bar{\mu}_T$ are function of γ_1^H, thus, I define a real function h whose support are feasible values of γ_1^H and is given by

$$h(\gamma_1^H) = \bar{\mu}_{CO} - \bar{\mu}_T$$

Note that for values γ_1^H where h is positive then $\bar{\mu}_{CO} > \bar{\mu}_T$ and for values that it is negative then $\bar{\mu}_T > \bar{\mu}_{CO}$. First, for feasible $\gamma_1^H \leq 2\rho_0$, function h is positive since, by definition, $\bar{\mu}_T$ is negative, and I already showed that $\bar{\mu}_{CO}$ is always positive. Thus, for feasible $\gamma_1^H \leq 2\rho_0$ then $\bar{\mu}_{CO} > \bar{\mu}_T$. Now, note that the sign of function h is determined by the sing of function g (I eliminate superscript H from γ_1^H to simplify notation)

$$g(\gamma_1) = \rho_0^2 + 2\rho_0\rho_1 + \gamma_1^2 - 2\gamma_1\left(\rho_0 + \frac{\rho_1}{2}\right)$$

Function g is a quadratic function that reaches a minimum at $\rho_0 + \frac{\rho_1}{2}$. Evaluating g at its minimum, one gets that

$$g(\rho_0 + \frac{\rho_1}{2}) = \rho_1^2\,(3/4 - \theta)$$

Therefore, as long as θ is less or equal to $3/4$, then $\bar{\mu}_{CO} > \bar{\mu}_T$. Now, for values of $\theta > 3/4$, γ^- and γ^+ are the roots of function g, thus, by definition of a quadratic function, for any feasible x between γ^- and γ^+, I get that $g(x)$ is negative. Therefore, when $\theta > 3/4$ and γ_1^H is between between $\max\{\gamma^-, 2\rho_0\}$ and γ^+, we have that $\bar{\mu}_{CO} < \bar{\mu}_T$. Lastly, I need to show that $\bar{\mu}_T < \mu_A$. This is true for any feasible γ_1^H. To see this, I assume that there exists a γ_1^H such that

$$O > \mu_A - \bar{\mu}_T$$

By definition of $\bar{\mu}_T$, one gets that

$$O > \mu_A - \bar{\mu}_T = \frac{1}{\mu_A}\left(\frac{\rho_0}{\gamma_1^H}\right)^2$$

which is strictly positive for any feasible γ_1^H. I find a contradiction, and, therefore, μ_A is always greater than $\bar{\mu}_T$ $\blacksquare$.